Learning What the Higgs Is Mixed With

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Fermilab - JUN 19, 2013

(work with R. Killick , H. E. Logan - arXiv:1305.7236)

OUTLINE

- Motivation for measuring $hhVV$ at ILC
- Measuring Higgs Couplings - LHC, ILC
- Parametrization of Couplings
- 3 Benchmark Models (BMs)
- $hhVV$ and hhh from di-Higgs rates at ILC
- M_{hh} as kinematic discriminant
- Caveats / Viability of BMs and Methodology
- Conclusions

Motivation

- We've entered the stage of measuring the properties of the 125 GeV resonance more precisely
- Through a long-term experimental program we hope to find out more about the nature of this particle and EWSB
- Many extensions of the SM involve the Higgs mixing with another scalar
- These scenarios can be tested by measuring coupling deviations or by direct searches
- Focus on the scenario where no new particles are observed

Motivation

- Measuring 3-pt couplings inform us about the degree of mixing, other scalar's contribution to EWSB and fermion coupling pattern
- Electroweak quantum numbers are not determined just by measuring the 3-pt couplings
- Easiest to see when the additional scalar does not contribute to EWSB or couple to fermions
- All couplings are modified by a common multiplicative factor that depends on the mixing angle

Motivation

- When it does contribute to EWSB it is not possible in general to disentangle the mixing angle and the EW quantum numbers in the 3-pt couplings
- $hhVV$ coupling depends on weak isospin and hypercharge and is accessible via electroweak-initiated di-Higgs production
- Measuring these at the LHC is extremely hard (details to follow)
- We propose to extract them from cross sections of at the following processes at the proposed ILC

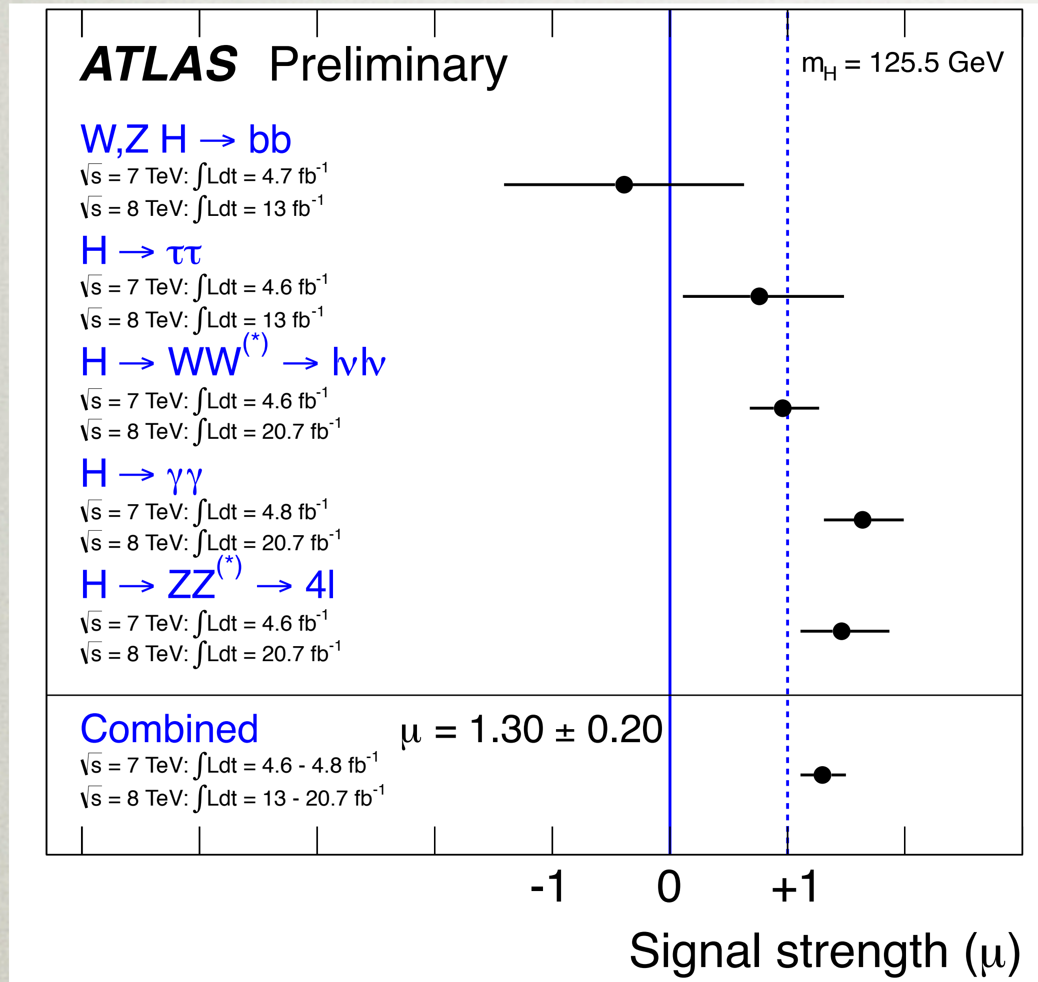
$e^+e^- \to Zhh$ (500 GeV) $e^+e^- \to \nu\bar{\nu}hh$ (1 TeV)

Motivation

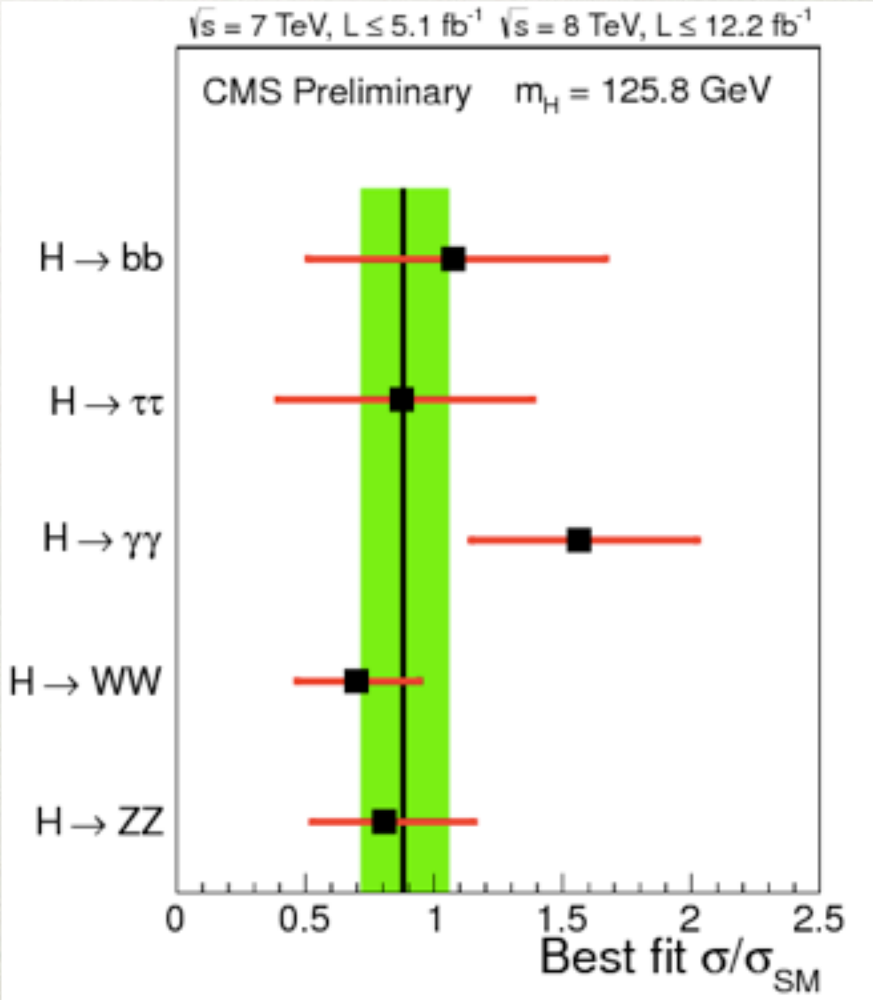
- These processes have been looked at in the past only to extract the Higgs self coupling (*hhh*)
- The two cross section measurements can be used to extract *hhVV* and *hhh* couplings simultaneously
- Allows us to distinguish between models where the Higgs mixes with scalars with different EW quantum numbers
- So the main goal of this work is to make a case for doing this hard measurement at the ILC in order to find out more about the EW quantum numbers of the scalar the Higgs mixes with

Measuring Higgs Couplings - LHC

- At the LHC what we measure are signal strengths (production x Branching Ratio)



Moriond 2013

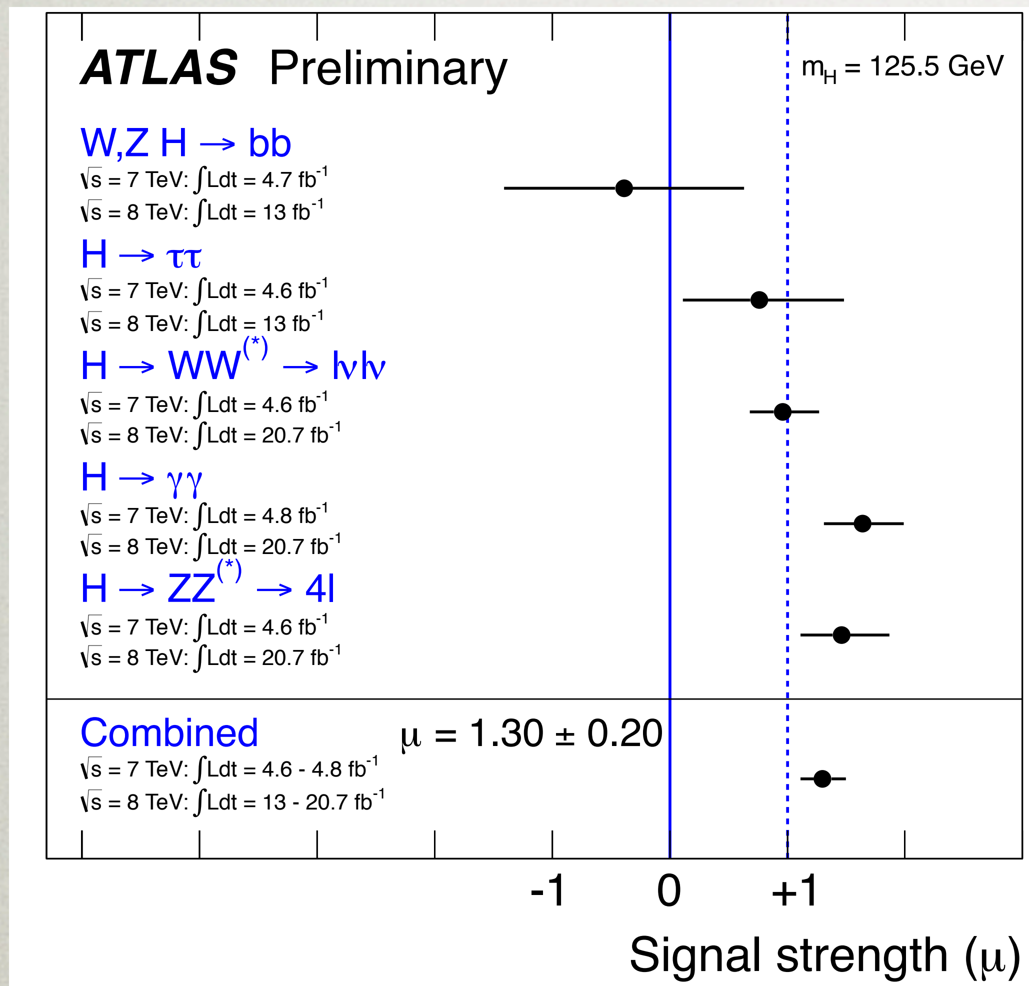


$\hat{\mu} = 0.88 \pm 0.21$

HCP 2012

Measuring Higgs Couplings - LHC

- Extracting Higgs couplings in a model independent way from the signal strength require global fits (too many parameters vs model dependence)



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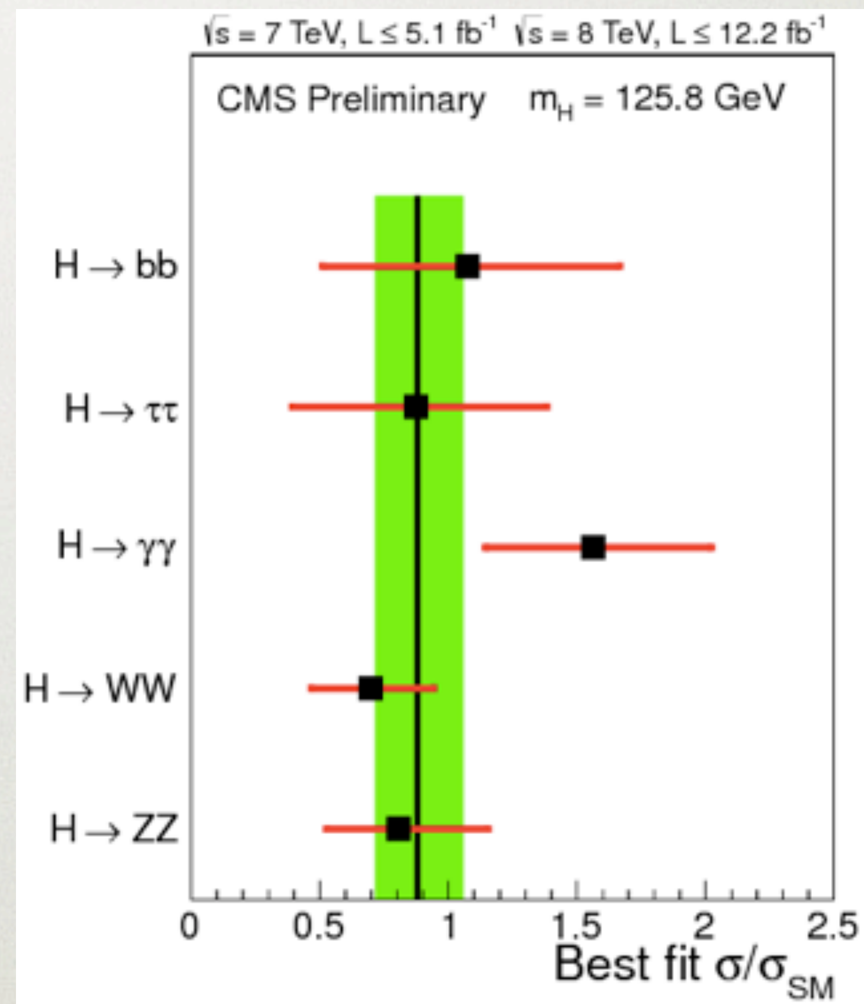
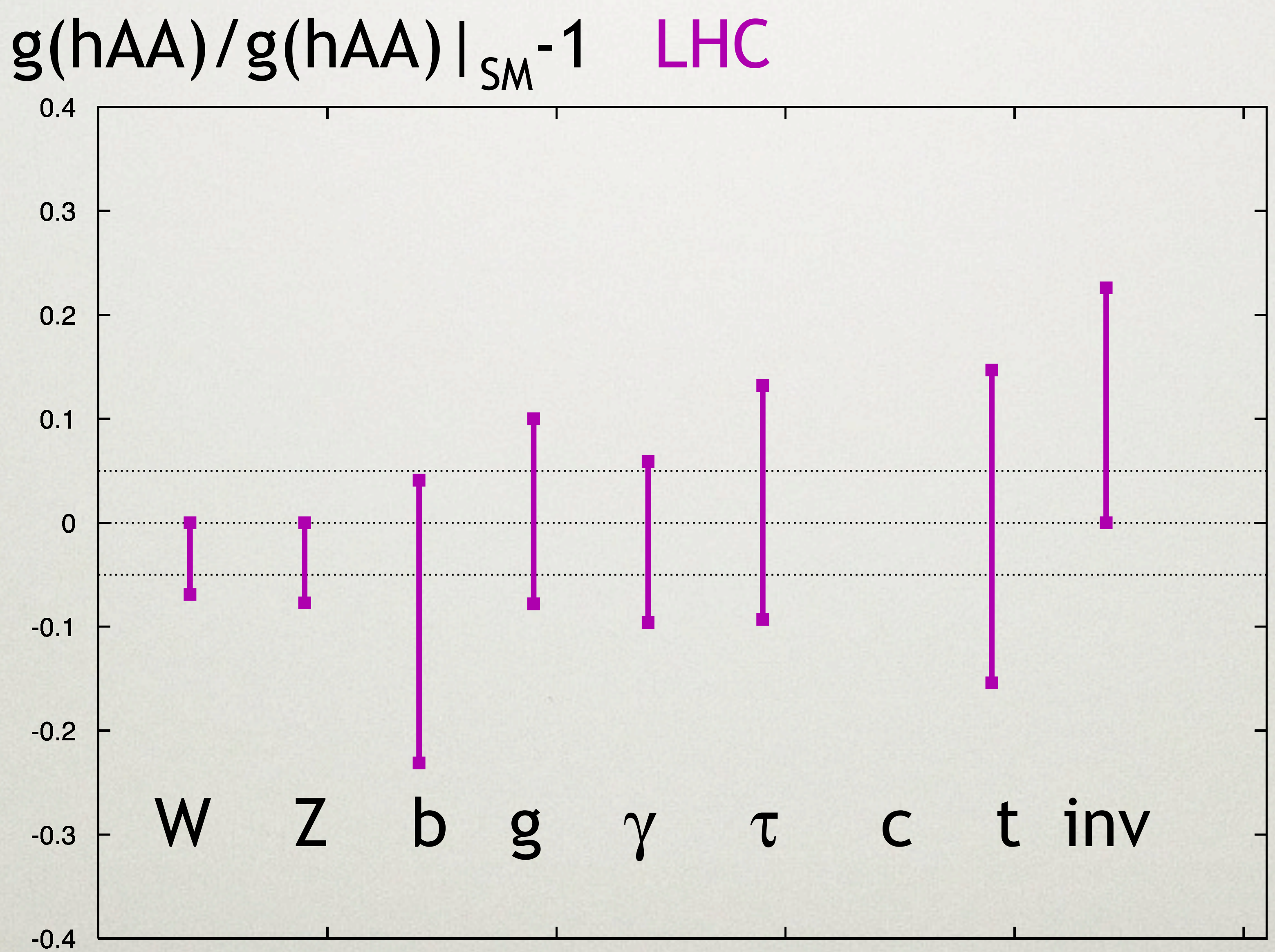


$\hat{\mu} = 0.88 \pm 0.21$

HCP 2012

Measuring Higgs Couplings - LHC

- Estimates for accuracy in Higgs coupling measurements with 300 inv fb of data (end of this decade)



M. Peskin, arXiv : 1208.5152

Measuring Higgs Couplings - LHC

- *hhVV* and *hhh* couplings are hard to measure because the cross sections for di-Higgs production are small
- At 14 TeV LHC
  pp -->h ~ 50 pb (gluon fusion)
  pp --> h h ~ 20 fb (gluon fusion)
  pp --> h h ~ 2 fb (VBF)

A. Djouadi, W. Kilian, M. Muhlleitner and P. Zerwas, Eur. Phys. J. C10 (1999) 45

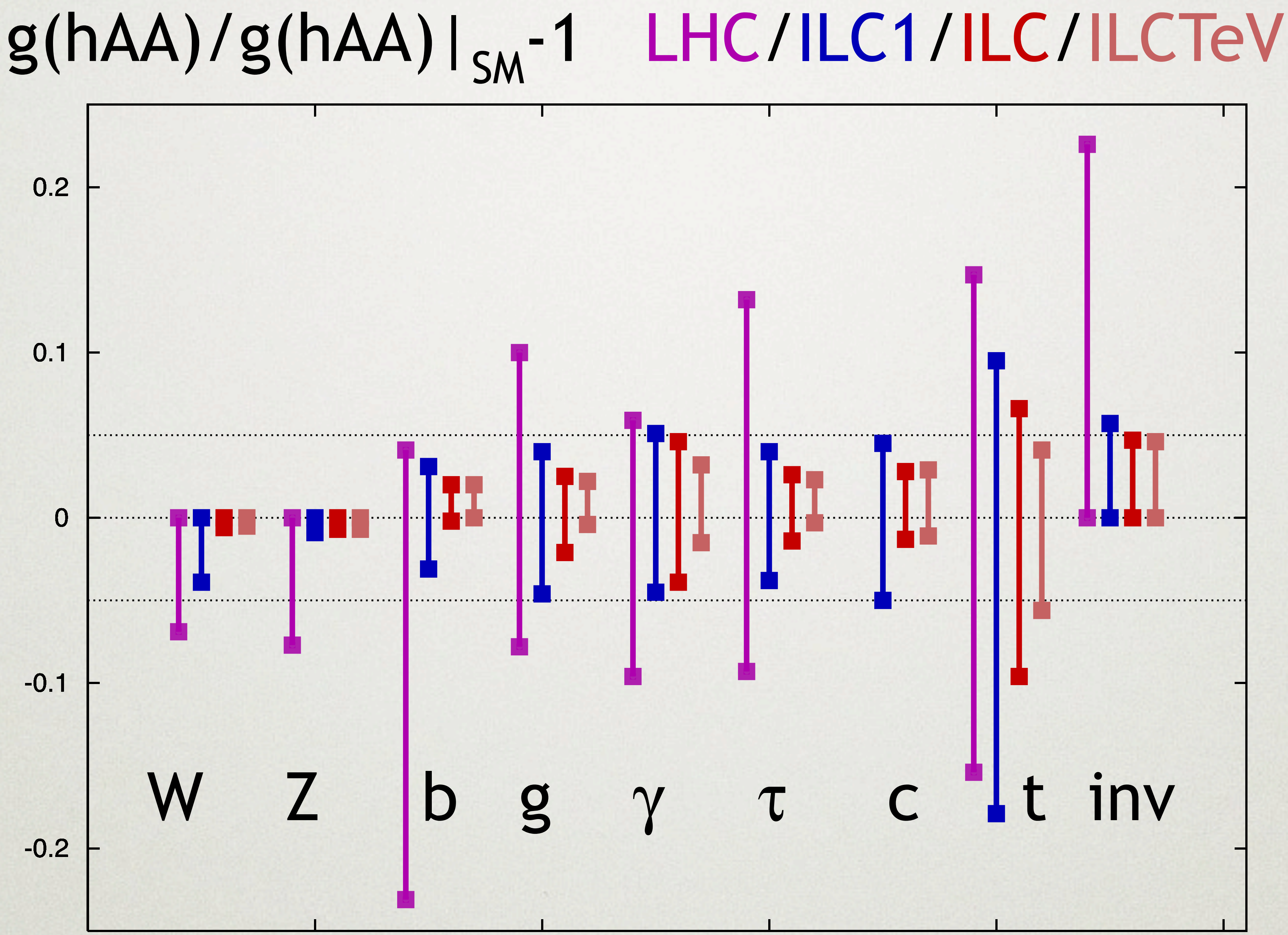
Building the ILC

- There are good reasons to do better even in channels that the LHC measures to ~10% precision
- A number of NP scenarios with a light Higgs and other particles (heavier than a TeV) can cause deviations smaller than that in one or more of the Higgs couplings (decoupling limit)
- In the absence of any other particles being discovered at the LHC, measuring the Higgs couplings more precisely is crucial
- Measuring these couplings more precisely is one of the main physics reasons to build a linear collider like the proposed ILC

Advantages

- e+ e- collisions have much smaller total cross sections (~100 nb as compared to ~100 mb)
- No pile up or hadrons from underlying event
- Z and W bosons are recognized easily even in hadronic decay modes
- Absolute branching ratios of the Higgs can be measured as the Higgs can be tagged when it recoils against the Z boson in $e^+e^- \to Zh$ at 250 GeV
- Combined with $\sigma(e^+e^- \to \nu\bar{\nu}h \to b\bar{b})$ at 250 and 500 GeV gives the Higgs width to 6%

LHC-ILC comparison



LHC - 14 TeV , 300 inv. fb

ILC1 - 250 GeV, 250 inv. fb

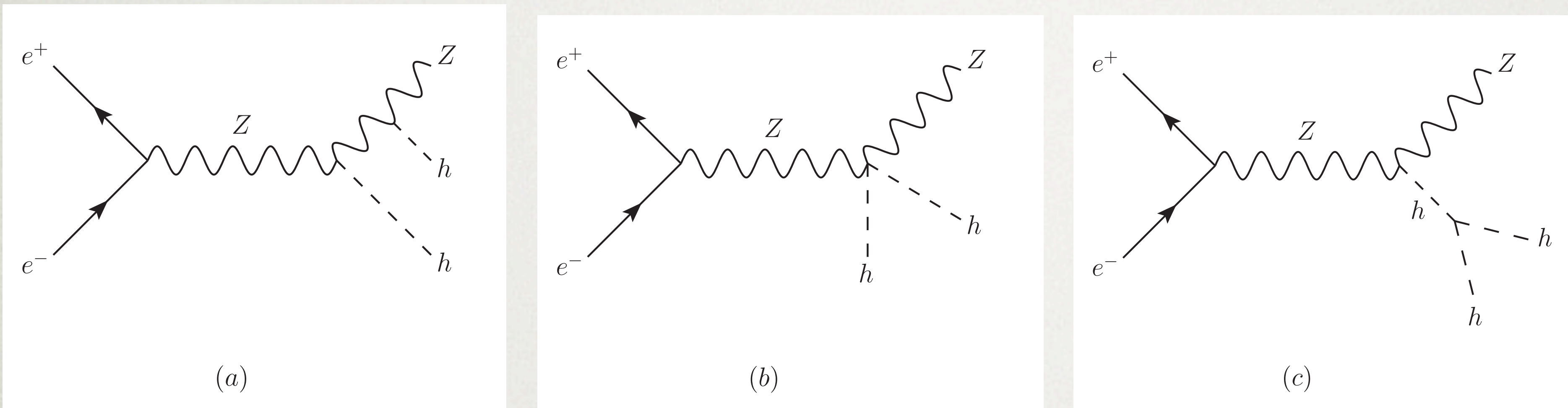
ILC - 500 GeV, 500 inv. fb

ILCTeV - 1 TeV, 1000 inv. fb

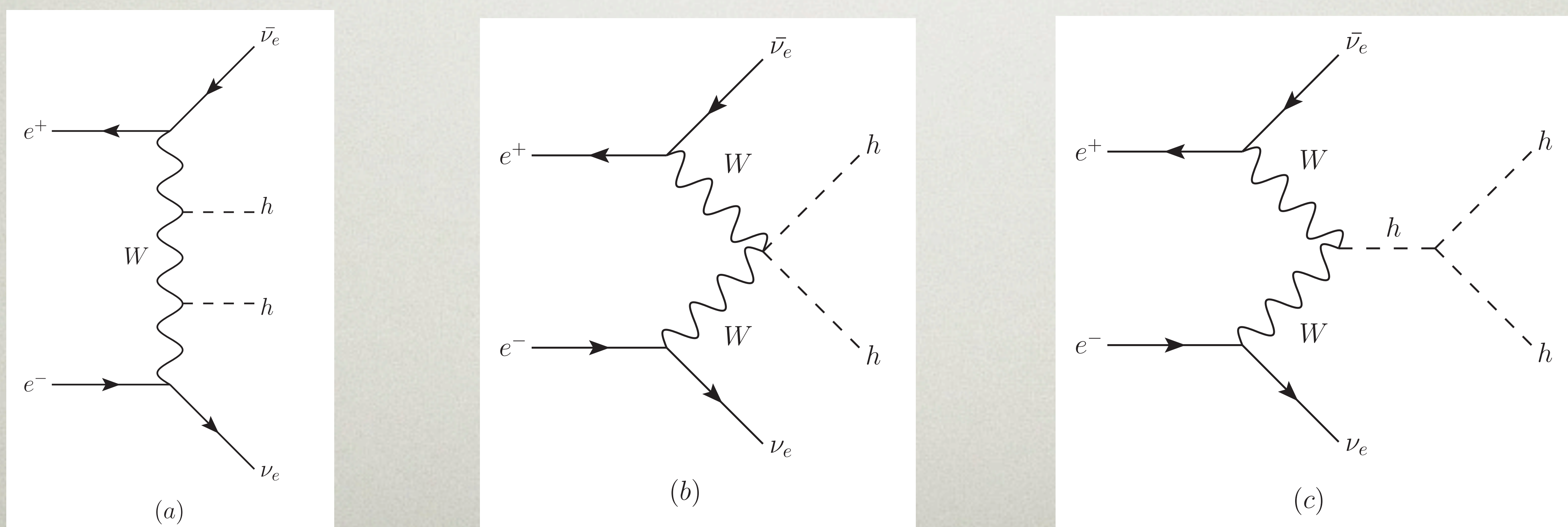
M. Peskin, arXiv : 1208.5152

Double Higgs production at ILC

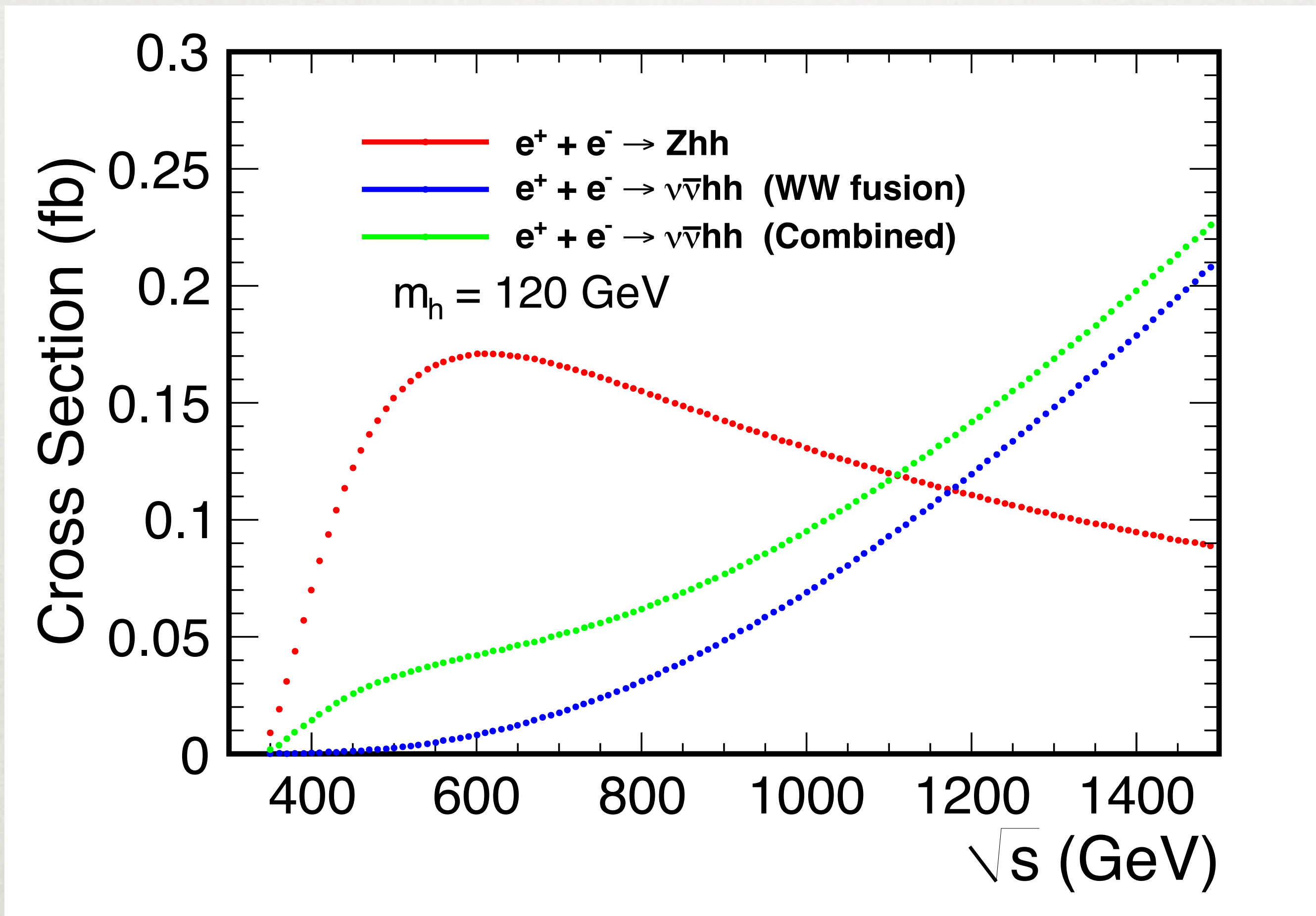
$$e^+e^- \to Zhh \quad (500\ \mathrm{GeV})$$



$$e^+e^- \to \nu\bar{\nu}hh \quad (1\ \mathrm{TeV})$$ WBF and $Z(\to \nu\bar{\nu})hh$



Double Higgs production at ILC



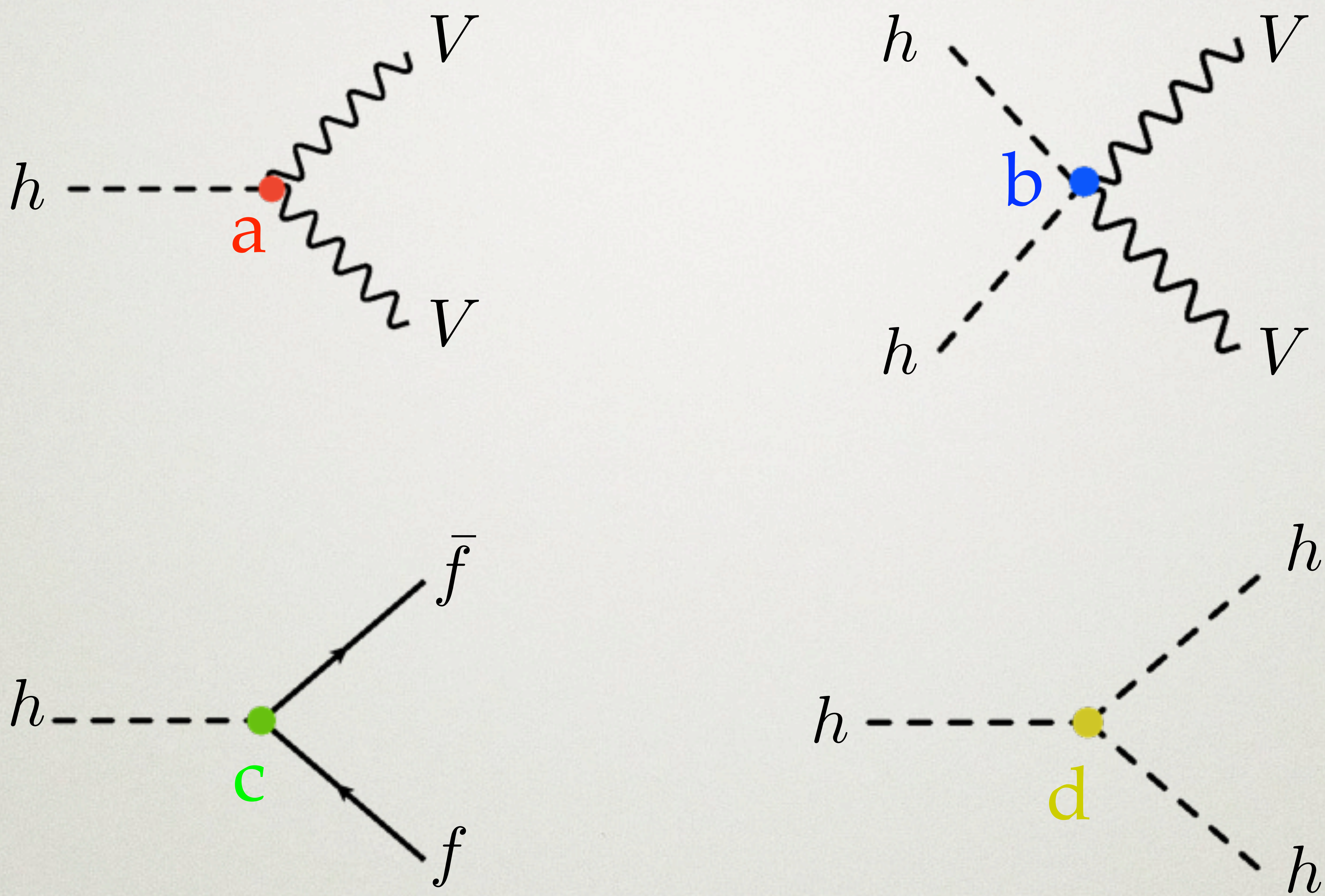
PARAMETRIZATION OF COUPLINGS

$$\mathcal{L} \supset k_V M_V^2 V_\mu^* V^\mu \left[1 + a_V \frac{2h}{v_{\rm SM}} + b_V \frac{h^2}{v_{\rm SM}^2}\right] - m_f \bar{f} f \left[1 + c_f \frac{h}{v_{\rm SM}}\right]$$

$$-\frac{1}{2} M_h^2 h^2 \left[1 + d_3 \frac{h}{v} + d_4 \frac{h^2}{4v^2}\right]$$

- In the SM, $a_i, b_i, c_i, d_i = 1$
- c_f can be different for each fermion
- $V = W$ or Z
- $k_W = 1$, $k_Z = 1/2$
- $(a_W, b_W) \neq (a_Z, b_Z)$ in models where custodial SU(2) is violated

Nomenclature



a,b,c,d are multiplicative factors by which the SM couplings are modified and V denotes the W or Z boson

COUPLING MODIFICATIONS

$$h = \phi \cos\theta - \chi \sin\theta.$$

- χ - real neutral component of general electroweak multiplet X
- If it doesn't couple to fermions or get a non-zero vev

$$a_W = a_Z \equiv a = \cos\theta, \qquad c_f \equiv c = \cos\theta.$$

- When X acquires a vev it contributes to masses of W & Z bosons
- Also leads to χ coupling with WW and ZZ

$$a \neq c$$

$$a_V = \cos\theta \sin\beta - \sqrt{b_V^\chi} \sin\theta \cos\beta, \qquad c = \frac{\cos\theta}{\sin\beta}$$

$$\sin\beta = v_\phi / v_{\rm SM}$$

$$b_W^\chi = 2\left[T(T+1) - \frac{Y^2}{4}\right], \qquad b_Z^\chi = Y^2.$$

SM Higgs T = 1/2,Y=1

COUPLING MODIFICATIONS

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$$\sin\beta = v_\phi / v_{\mathrm{SM}}$$

$$b_W^\chi = 2\left[T(T+1) - \frac{Y^2}{4}\right], \qquad b_Z^\chi = Y^2.$$

(SM Higgs T = 1/2,Y=1) $\quad b_W^\phi = b_Z^\phi = 1$

- Note that *a* and *c* depend on 3 parameters (b_V^χ, $\cos\theta$, $\sin\beta$)
- Thus b_V^χ can't be extracted by just measuring *a* and *c*

COUPLING MODIFICATIONS

- $hhVV$ couplings are modified by $\chi\chi VV$ couplings

$$\chi\chi W_\mu^+ W_\nu^- \ : \ i\frac{g^2}{2} b_W^\chi g_{\mu\nu}, \quad \chi\chi Z_\mu Z_\nu \ : \ i\frac{g^2}{2c_W^2} b_Z^\chi g_{\mu\nu}$$

- After mixing

$$b_V = \cos^2\theta + b_V^\chi \sin^2\theta.$$

- *X* does not carry vev
- *a* or *c* determine the mixing angle, *b* can be used to determine the quantum numbers

COUPLING MODIFICATIONS

- $hhVV$ couplings are modified by $\chi\chi^{VV}$ couplings

$$\chi\chi W^+_\mu W^-_\nu \; : \; i\frac{g^2}{2} b^\chi_W g_{\mu\nu}, \quad \chi\chi Z_\mu Z_\nu \; : \; i\frac{g^2}{2c_W^2} b^\chi_Z g_{\mu\nu}$$

- After mixing

$$b_V = \cos^2\theta + b^\chi_V \sin^2\theta.$$

- *X* carries vev
- *a*, *b* and *c* determine mixing angle, vev of *X* and its electroweak quantum numbers

COUPLING MODIFICATIONS

- $hhVV$ couplings are modified by $\chi\chi VV$ couplings

$$\chi\chi W^+_\mu W^-_\nu \,:\; i\frac{g^2}{2} b^\chi_W g_{\mu\nu}, \quad \chi\chi Z_\mu Z_\nu \,:\; i\frac{g^2}{2c_W^2} b^\chi_Z g_{\mu\nu}$$

- After mixing

$$b_V = \cos^2\theta + b^\chi_V \sin^2\theta.$$

- Models that preserve a custodial SU(2) have $b_W = b_Z$
- $b_V > 1$ is possible when T > 1/2

BENCHMARK MODELS

- We will assume the benchmark models preserve custodial SU(2)
- Additional scalars do not couple to fermions to avoid constraints from FCNC
- Additional scalar(s) carry zero or small vev
- The models differ only in the value of b ($a = 0.9$, $d = 1$)

BENCHMARK MODELS

- $\boxed{\text{I : SM + Real Singlet Scalar (a= c = 0.9, b = .81,d =1)}}$

$$h = \phi \cos\theta - s \sin\theta$$

$$b_W = b_Z = \cos^2\theta = a^2$$

(true irrespective of the singlet vev)

- $\boxed{\text{II : SM + Additional Doublet (a = 0.9, b = 1, d = 1)}}$

Type 1 2HDM where the doublet Φ_2 has small vev

CP-conserving potential with softly broken Z2

$$\lambda_6 = \lambda_7 = 0$$

$$\begin{aligned}\mathcal{V}_{\text{gen}} =& m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\ &+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\ &+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \left[\lambda_6 \left(\Phi_1^\dagger \Phi_1\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right)\right]\left(\Phi_1^\dagger \Phi_2\right) + \text{h.c.} \right\}.\end{aligned}$$

BENCHMARK MODELS

- I : SM + Real Singlet Scalar (a= c = 0.9, b = .81,d =1)

$$h = \phi \cos\theta - s \sin\theta$$

$$b_W = b_Z = \cos^2\theta = a^2$$

(true irrespective of the singlet vev)

- II : SM + Additional Doublet (a = 0.9, b = 1, d = 1)

Consider a Type 1 2HDM where the doublet Φ_2 has small vev

$$h = \phi_1 \cos\theta - \phi_2 \sin\theta$$

CP-conserving potential with softly broken Z2

$$b_W = b_Z = \cos^2\theta + \sin^2\theta = 1$$

BENCHMARK MODELS

- III : Georgi-Machacek model   (a = 0.9, b = 1.32,d=1)
- Contains the SM doublet along with a complex triplet (Y=2) and a real triplet (Y=0)
- Together they can be arranged so as to preserve custodial SU(2)

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

$$\langle \Phi \rangle = \frac{v_\phi}{\sqrt{2}} 1_{2\times 2} \qquad \langle X \rangle = v_\chi 1_{3\times 3}$$

$$v_{\rm SM}^2 = v_\phi^2 + 8 v_\chi^2$$

- Note that each of those triplets taken individually with SM would violate custodial SU(2)

BENCHMARK MODELS

- The model contains two custodial SU(2) singlets that can mix to produce the observed resonance

$$H_1^0 = \phi \qquad\qquad H_1^{0\prime} = \sqrt{\frac{2}{3}}\chi^{0,r} + \frac{1}{\sqrt{3}}\xi^0$$

$$h = H_1^0 \cos\theta - H_1^{0\prime}\sin\theta$$

$$b = \cos^2\theta + \frac{8}{3}\sin^2\theta$$

- For small X vevs the following approximation can be made

$$b_W = b_Z = a^2 + \frac{8}{3}(1 - a^2)$$

- For a = 0.9 this yields b = 1.32

Measuring the $hhVV$ Coupling

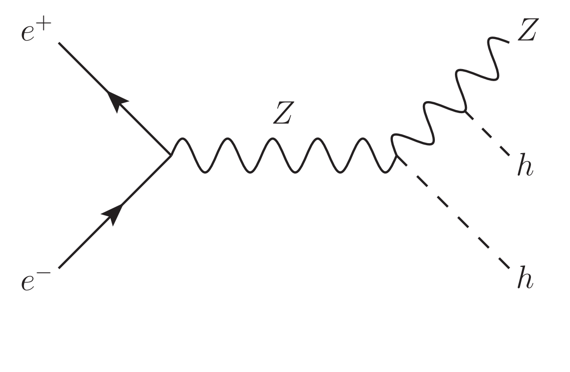
- Scenario : LHC + 250 GeV ILC data point to the Higgs mixing with another scalar that doesn't couple singly to fermions and whose gauge boson couplings are negligible

$$a = c = 0.9$$

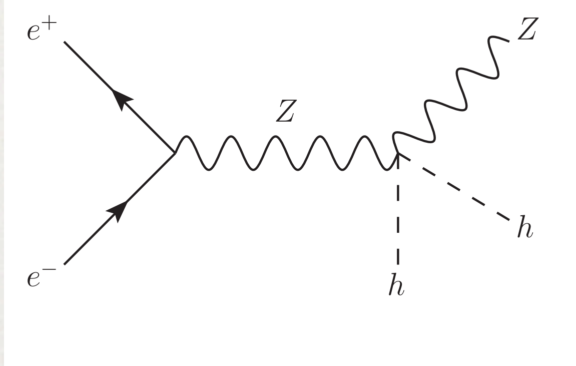
- The production rate would be scaled by a factor of 0.81 but all the BRs would stay the same
- The 250 GeV ILC measurement of $e^+e^- \to Zh$ would yield a to a precision of $\Delta a/a = 1.3\%$ with 250 inv. fb of data (~ 3 yrs)

Double Higgs production at ILC

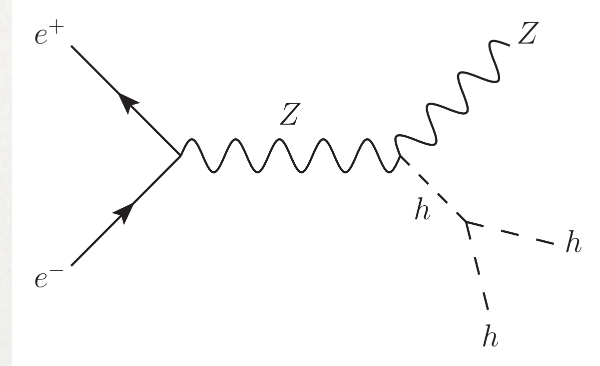
$e^+e^- \to Zhh$ (500 GeV)



(a)

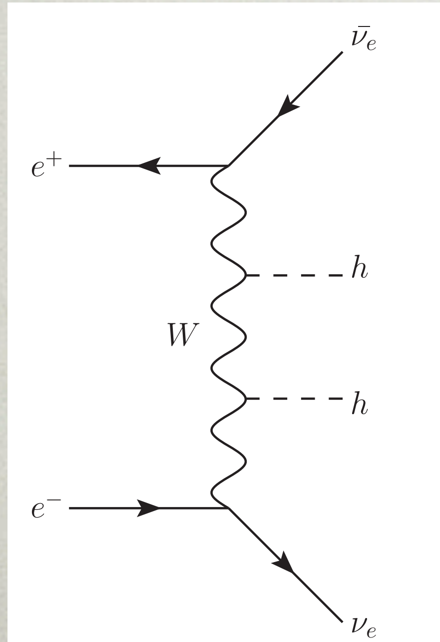


(b)



(c)

$e^+e^- \to \nu\bar{\nu}hh$ (1 TeV) WBF and $Z(\to \nu\bar{\nu})hh$



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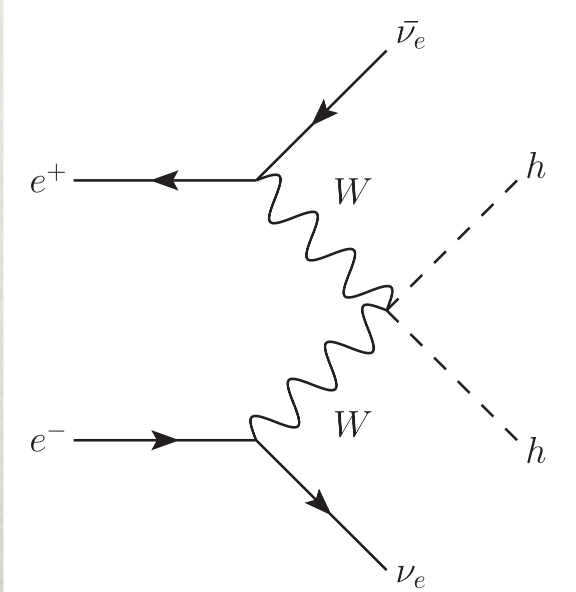
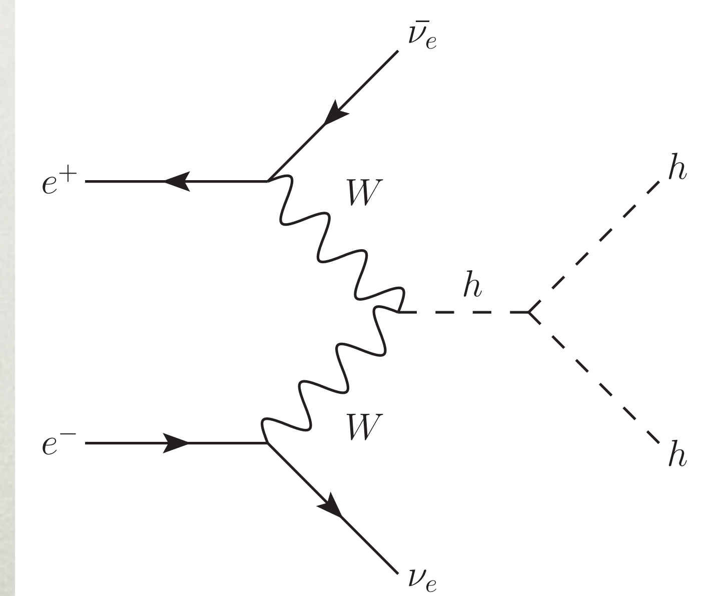


(b)



(c)

Double Higgs production at ILC

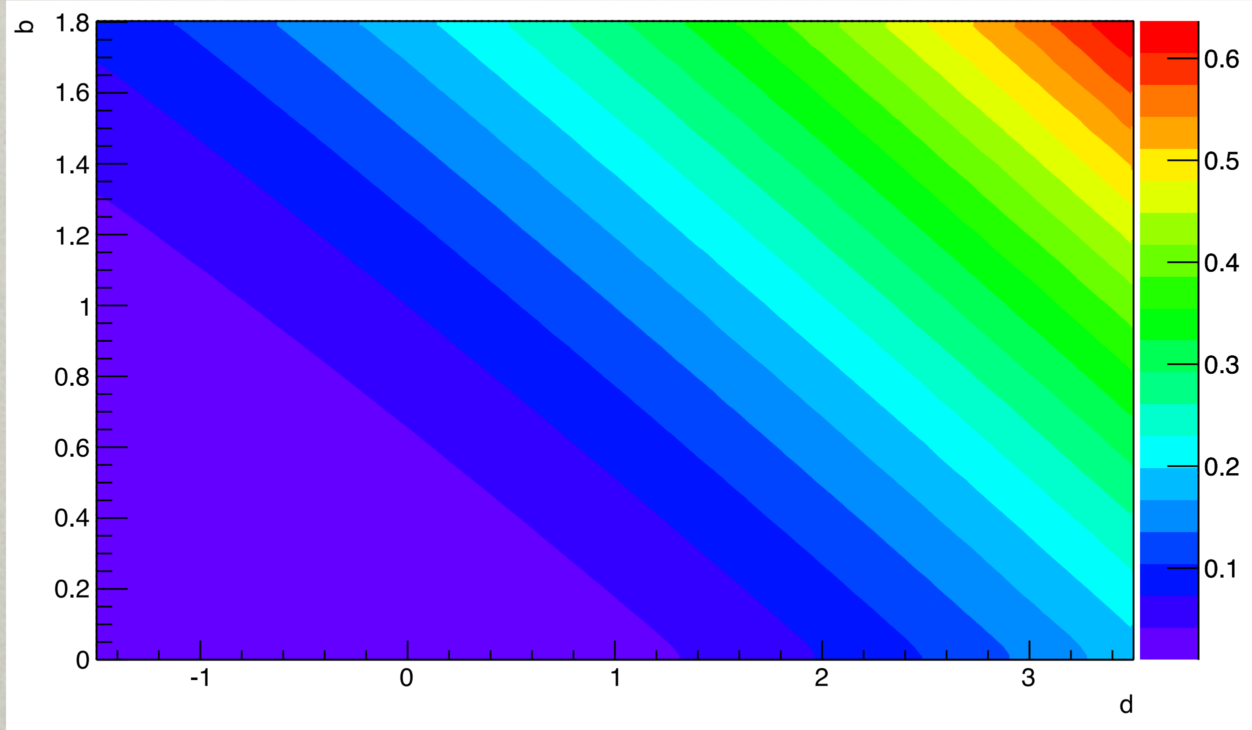
- We calculate the cross sections using CalcHEP and MG5 for di-higgs production for the SM and the three benchmark models assuming $a = 0.9$ and $d = 1$ (unpolarized beams)
- We choose our BM pts such that the additional heavy states are beyond the kinematic reach of the ILC and their contribution to the cross sections is negligible

Model	b	$\sigma^{500}(Zhh)$	$\sigma^{1000}(Zhh)$	$\sigma^{1000}(\mathrm{WBF})$
Singlet	0.81	0.11 fb	0.082 fb	0.041 fb
Doublet	1	0.14 fb	0.11 fb	0.027 fb
GM	1.32	0.19 fb	0.18 fb	0.090 fb
SM	1	0.16 fb	0.12 fb	0.071 fb

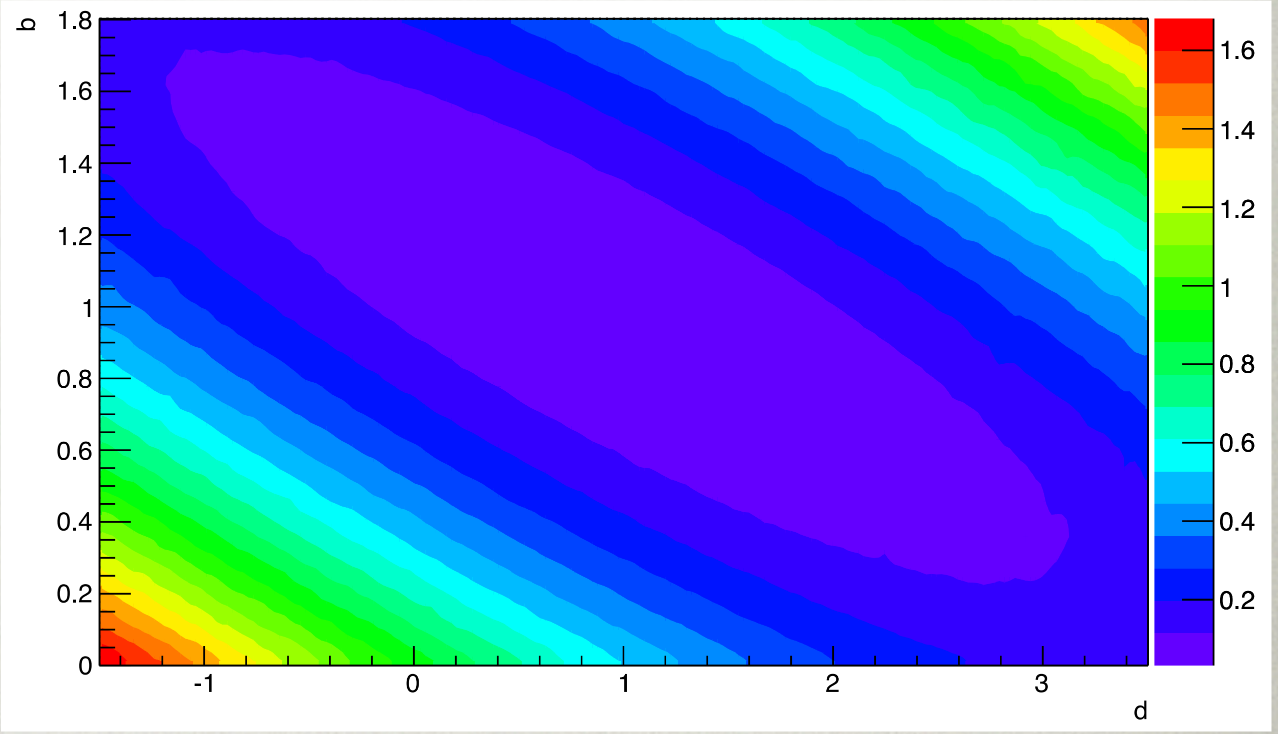
Extracting b and d

- The cross sections for the two processes depend differently on b and d
- This dependence also varies with the CoM energy

$e^+e^- \to Zhh$ (500 GeV)



$e^+e^- \to \nu\bar{\nu}hh$ (1 TeV)



Contour Plots of Cross sections (in fb)

Extracting b and d

- Measurements of $e^+e^- \to Zhh$ at 500 GeV and $e^+e^- \to \nu\bar{\nu}hh$ at 1 TeV can be used to fit for b and d
- We can compute the two cross sections in terms of our effective lagrangian for $a =$ 0.9 while varying b and d
- Next we can plot 68% and 95% CL chi sq plots for each Benchmark Model

$$\chi^2(b,d) = \sum_{i=1,2} \frac{(\sigma_i(b,d) - \sigma_{BM,i})^2}{\Delta\sigma_{BM,i}^2}$$

Extracting b and d

$$\chi^2(b,d) = \sum_{i=1,2} \frac{(\sigma_i(b,d) - \sigma_{BM,i})^2}{\Delta\sigma^2_{BM,i}}$$

- We use cross section uncertainties from the ILC Large Detector Study for the ILC Detailed Baseline Design (DBD) Report
- These uncertainties are scaled appropriately for the Benchmark Models as their cross sections are different from the SM

Model	b	$\Delta\sigma/\sigma(Zhh, 500\text{ GeV})$	$\Delta\sigma/\sigma(\nu\nu hh, 1\text{ TeV})$
Singlet	0.81	38%	32%
Doublet	1	32%	42%
GM	1.32	24%	18%
SM	1	27%	23%

Measuring b and d

- Account for different selection efficiencies for (Z--> v v) hh and WBF at 1 TeV by scaling the Zhh process to get the relative efficiency of 11%

Model	b	$\Delta\sigma/\sigma(Zhh, 500\text{ GeV})$	$\Delta\sigma/\sigma(\nu\nu hh, 1\text{ TeV})$
Singlet	0.81	38%	32%
Doublet	1	32%	42%
GM	1.32	24%	18%
SM	1	27%	23%

- Beam Polarisation : P(e-,e+) : (-0.8, +0.3) at 500 GeV

  and (-0.8,0.2) at 1 TeV , Int. Lum. = 2000 inv. fb
- Change in relative contribution of each feynman diagram due to kinematic cuts is beyond the scope of this work

Measuring *b* and *d*

- Relative uncertainty increases for Singlet and Doublet Benchmark model and decreases for GM as one would expect from the table of cross sections

Model	b	$\Delta\sigma/\sigma(Zhh, 500\text{ GeV})$	$\Delta\sigma/\sigma(\nu\nu hh, 1\text{ TeV})$
Singlet	0.81	38%	32%
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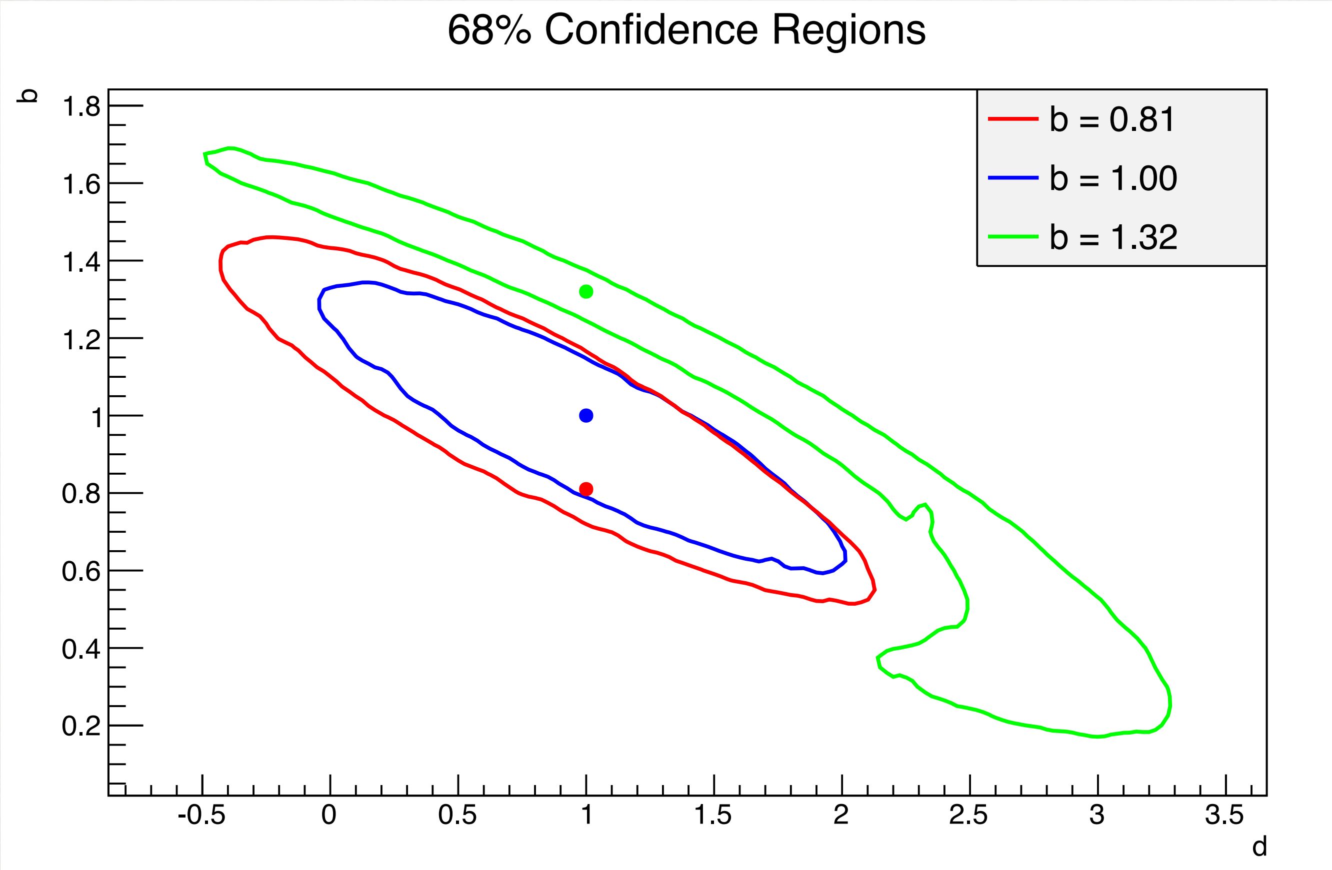
Model	b	$\sigma^{500}(Zhh)$	$\sigma^{1000}(Zhh)$	$\sigma^{1000}(\text{WBF})$
Singlet	0.81	0.109 fb	0.0815 fb	0.0411 fb
Doublet	1	0.136 fb	0.113 fb	0.0273 fb
GM	1.32	0.188 fb	0.183 fb	0.0901 fb
SM	1	0.157 fb	0.119 fb	0.0712 fb

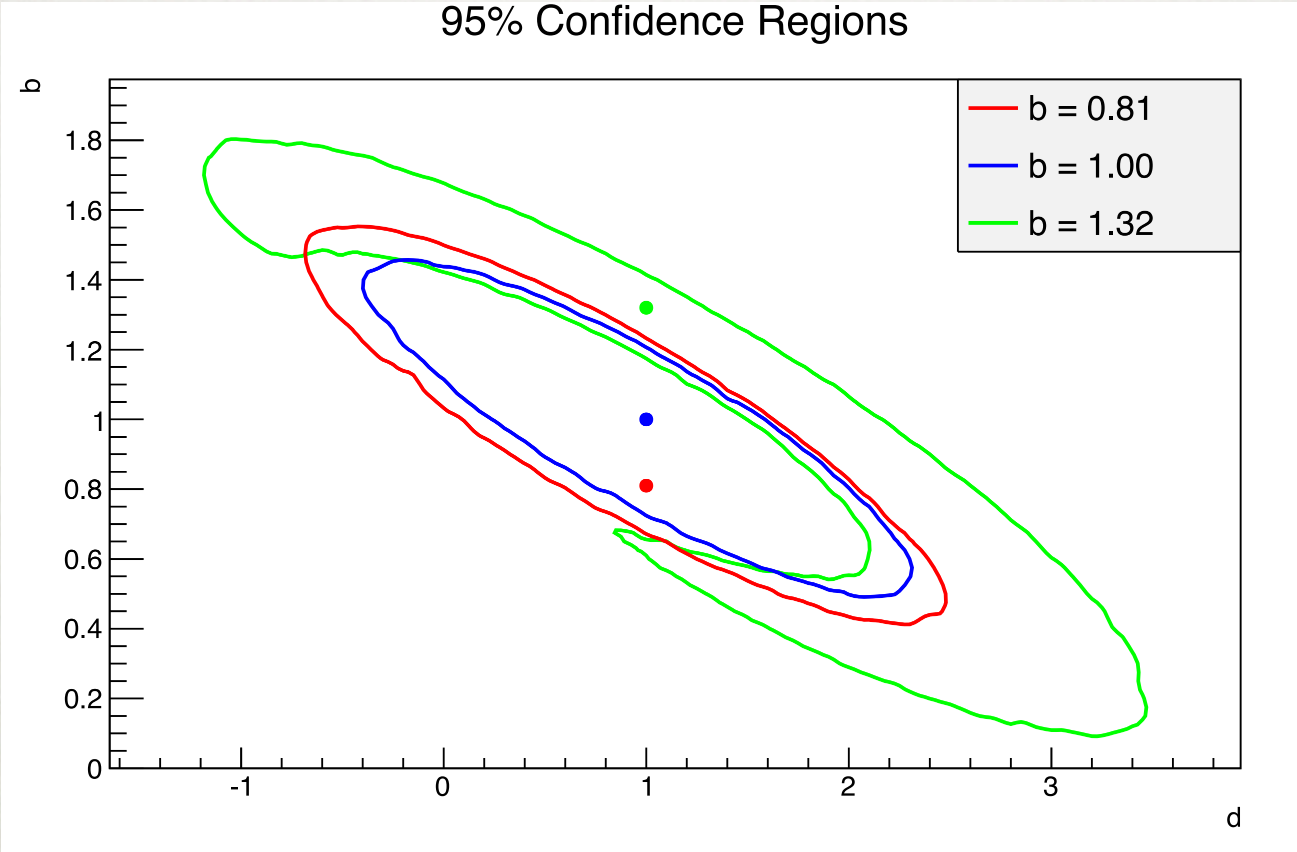
Fit Results

- GM model can be distinguished from singlet and doublet benchmarks at 68% CL (chi sq. = 2.28)



Fit Results

- Overlap at 95% CL is minimal (chi sq. = 5.99)

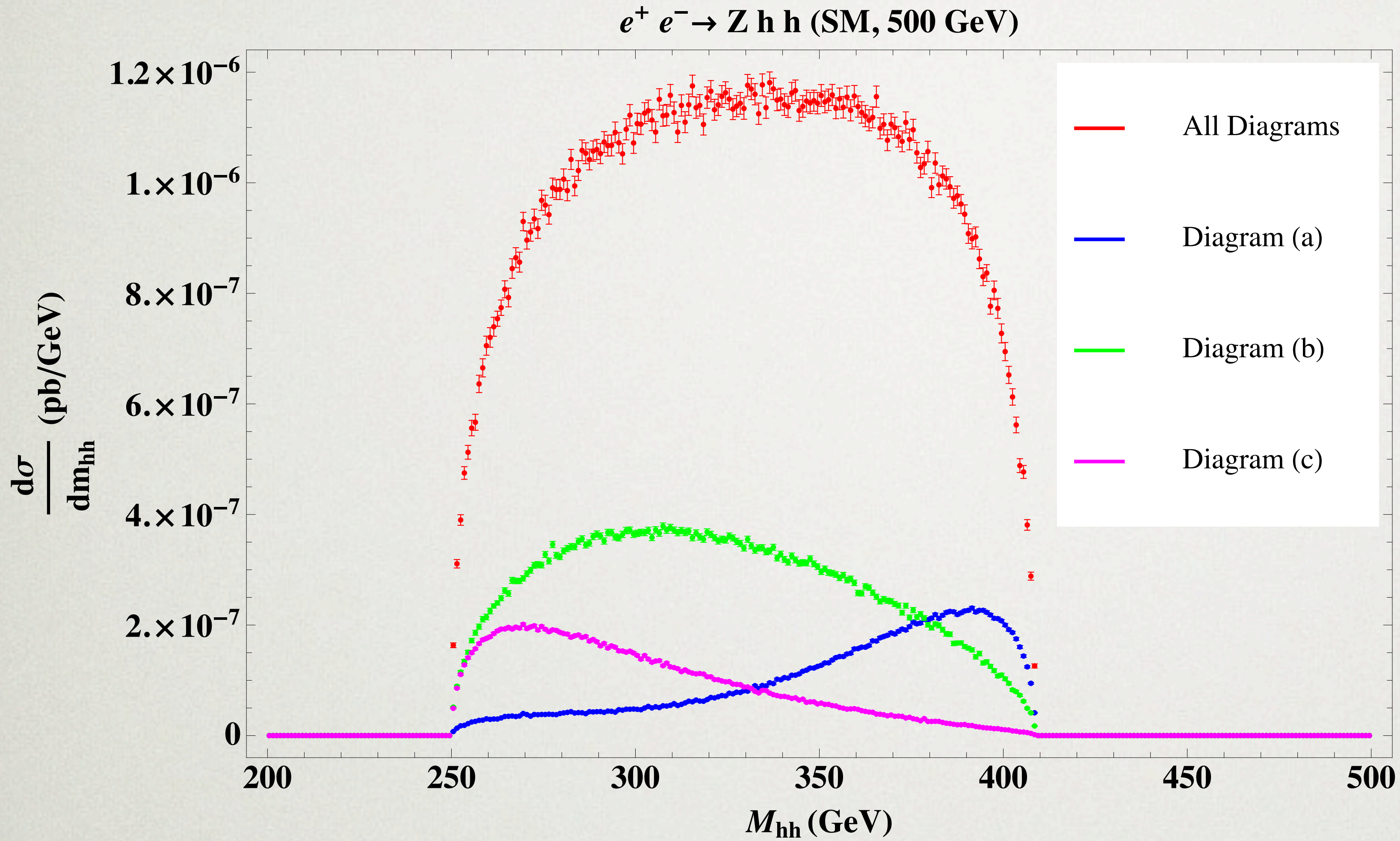


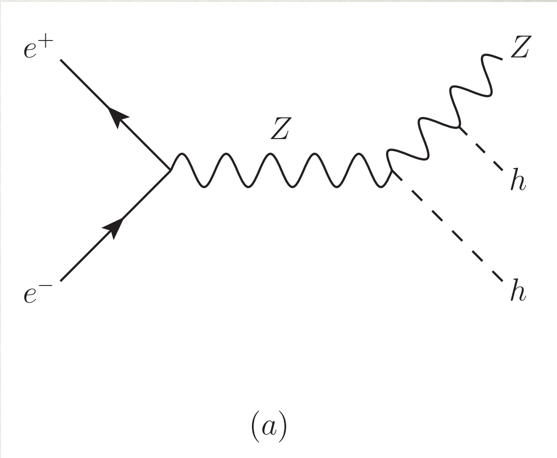
- Crescent shape due to WBF cross section not being monotonic in *b*

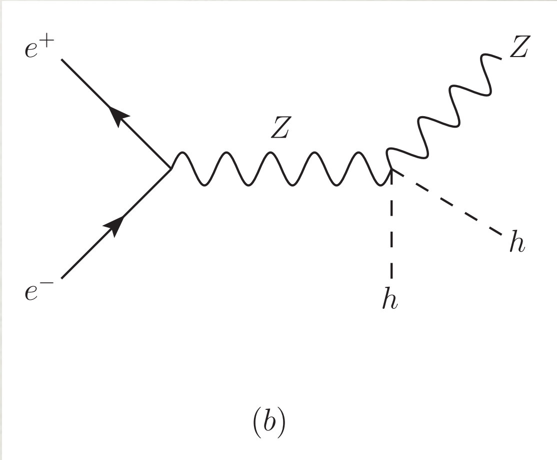
Measuring *b* and *d*

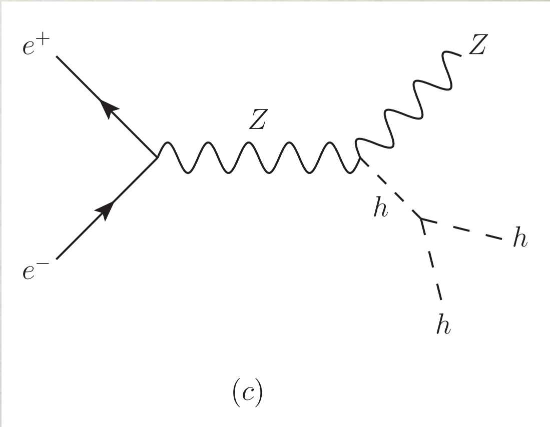
- The DBD report assumed a Higgs mass of 120 GeV and considered the channel where higgs decays to bottom quark pairs
- At 125 GeV this would reduce the cross sections by about 20%
- The lost precision can be regained by including the $hh \to WWb\bar{b}$

M_{hh} as Kinematic Discriminant (Zhh)





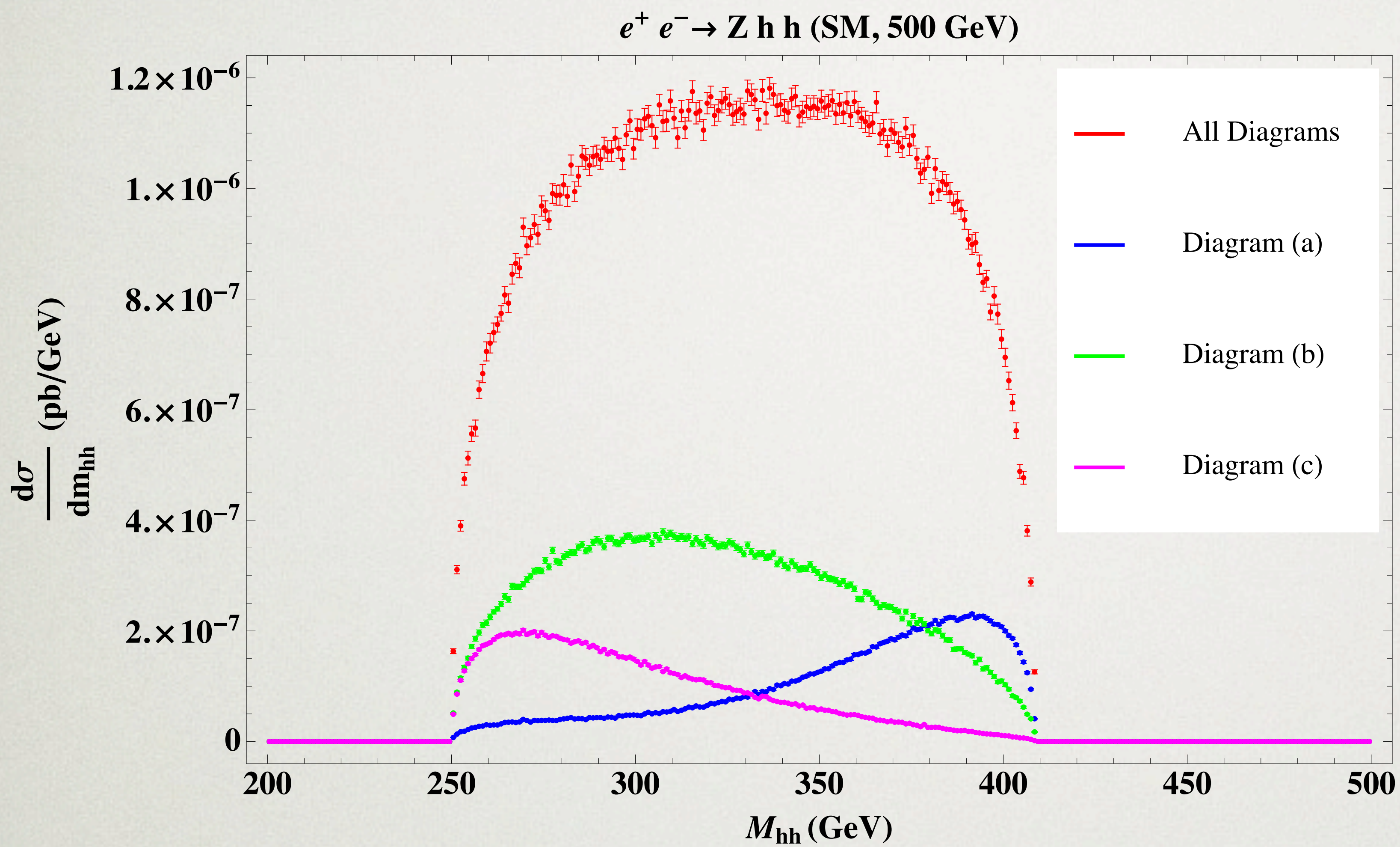


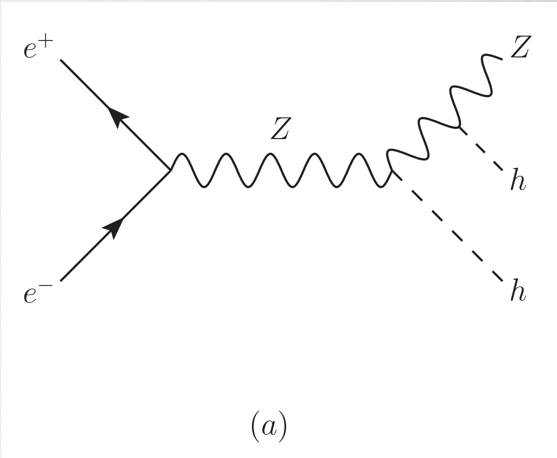


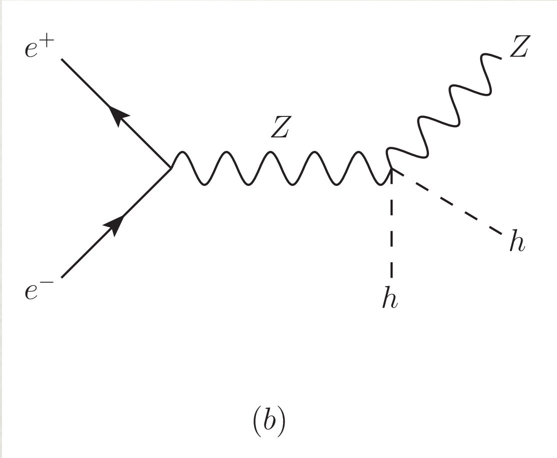
- ILD collaboration is exploring improving sensitivity to d by weighting events based on M_{hh}

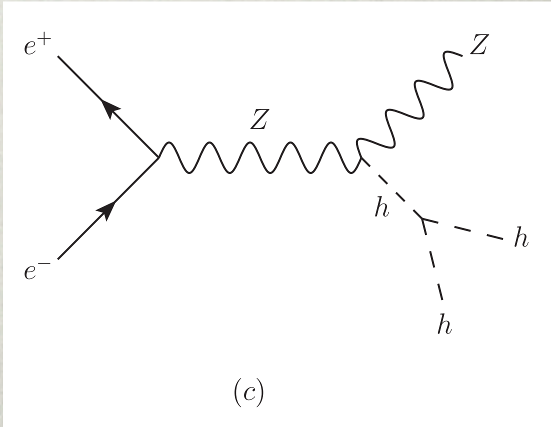
J.Tian, talk at LCWS2012, http://www.uta.edu/physics/lcws12/

M_{hh} as Kinematic Discriminant (Zhh)





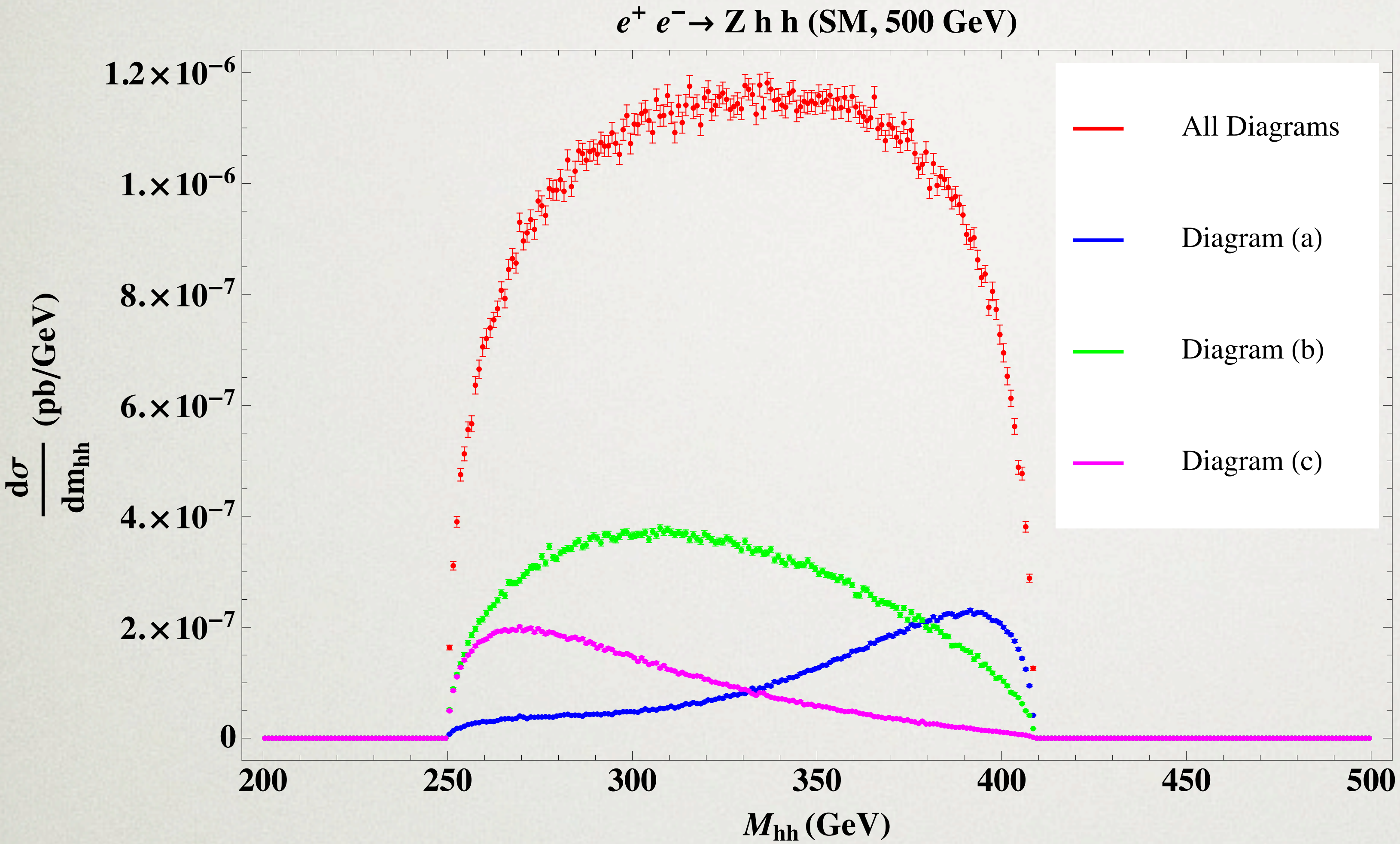


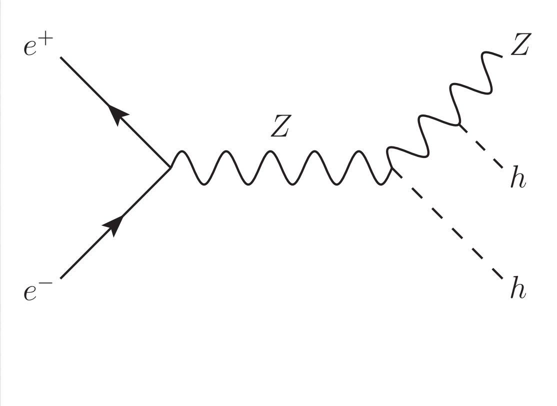


- Improves precision on d by 10% at 500 GeV and 1 TeV for the SM case ($a = b = 1$)

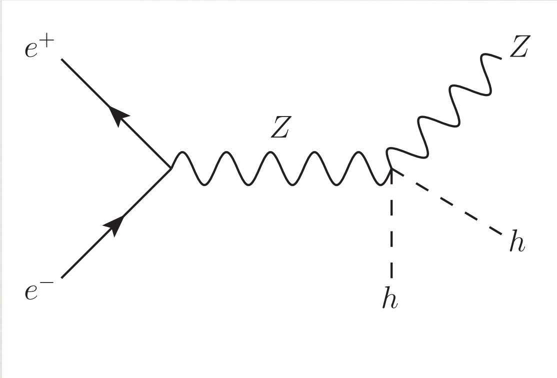
K. Fujii, Talk at Higgs Snowmass Workshop 2013, http://physics.princeton.edu/snowmass

M_{hh} as Kinematic Discriminant (Zhh)

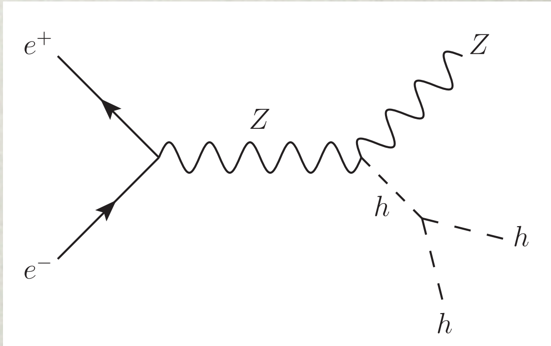




(a)



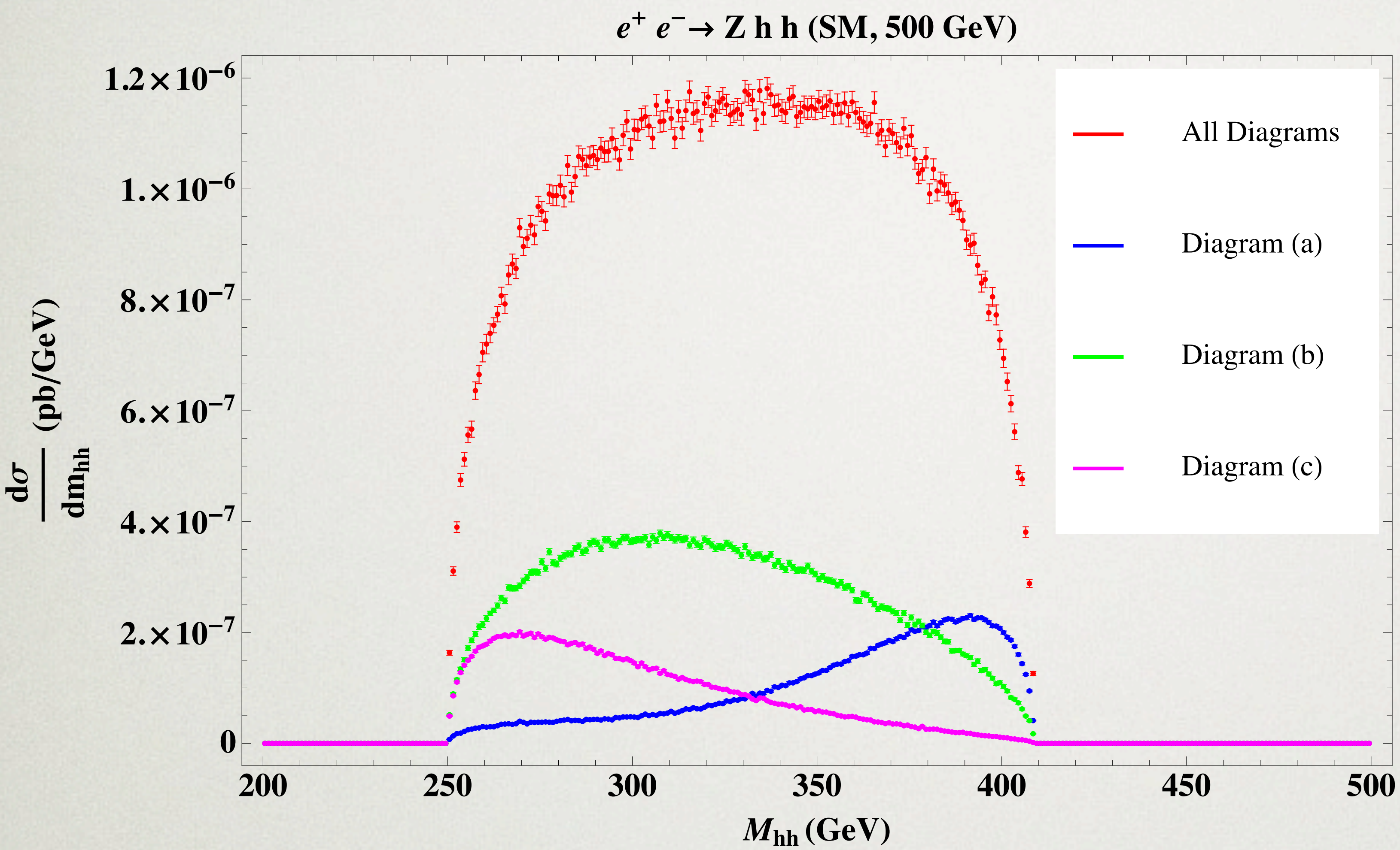
(b)

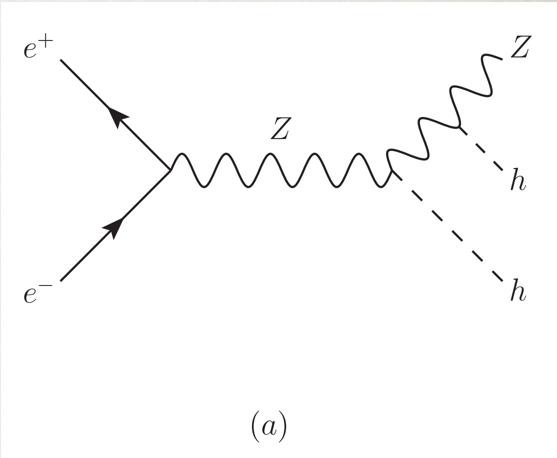


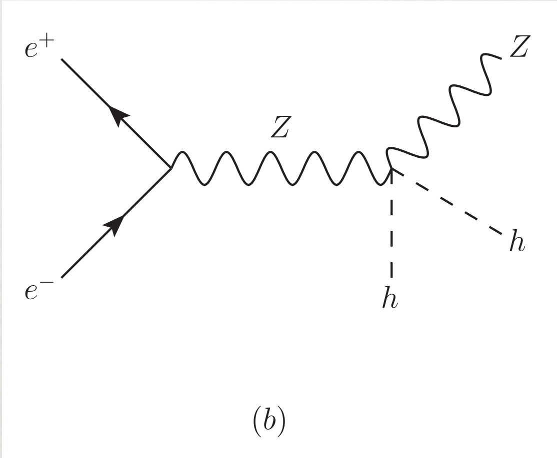
(c)

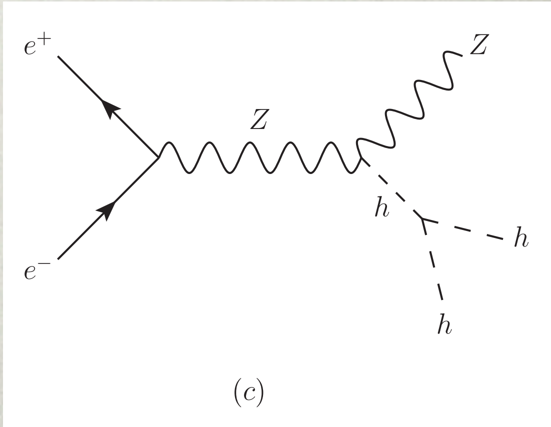
- Method can be adapted to improve extraction of b and d
- Significant contribution from interference
- Contribution from (c) is higher at lower M_{hh} and from (b) has a broader M_{hh} dependence

M_{hh} as Kinematic Discriminant (Zhh)



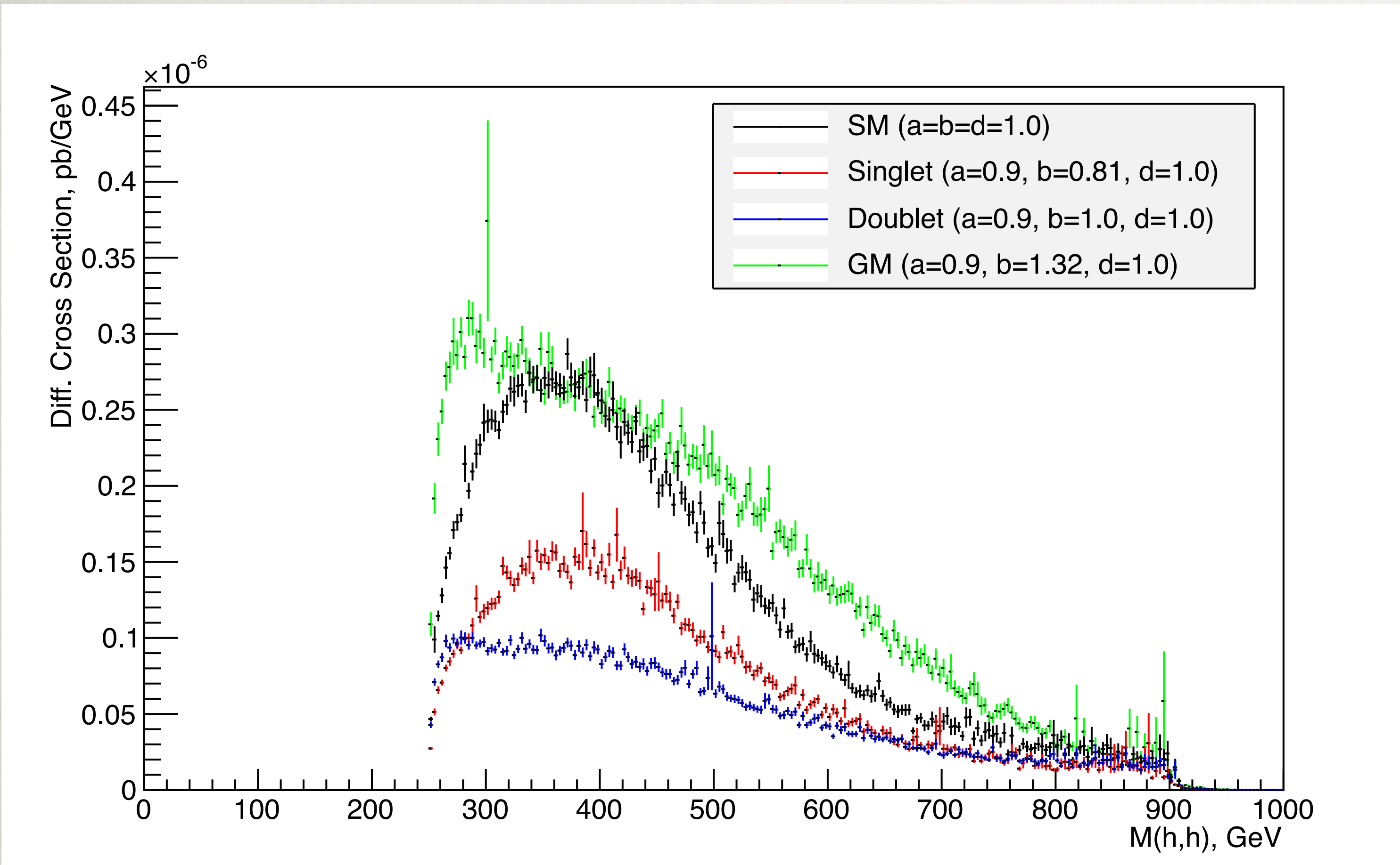


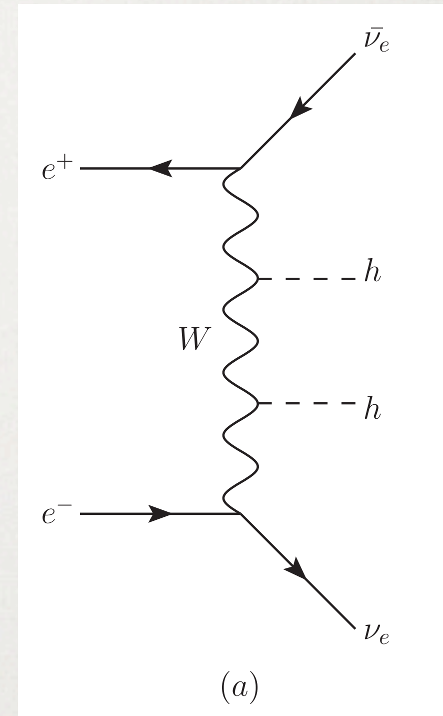


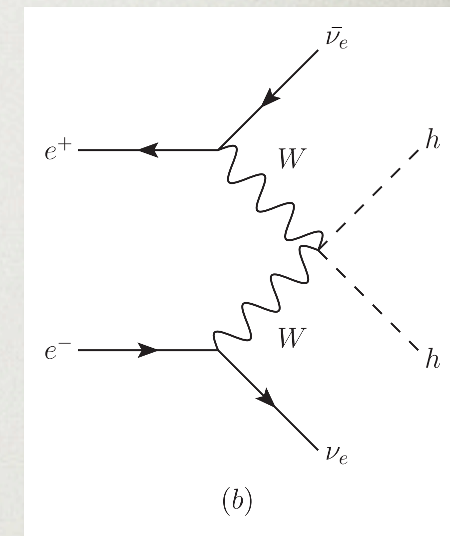


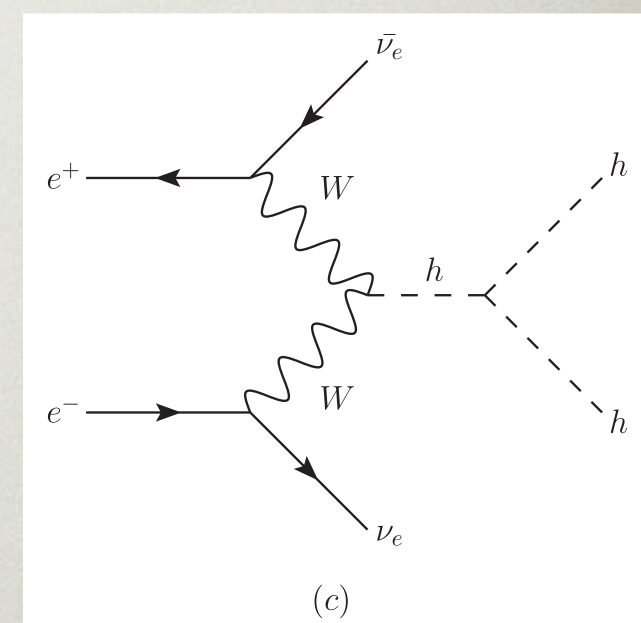
- Contribution from d is highest at low M_{hh} and from b has a broader M_{hh} dependence

WBF at 1 TeV



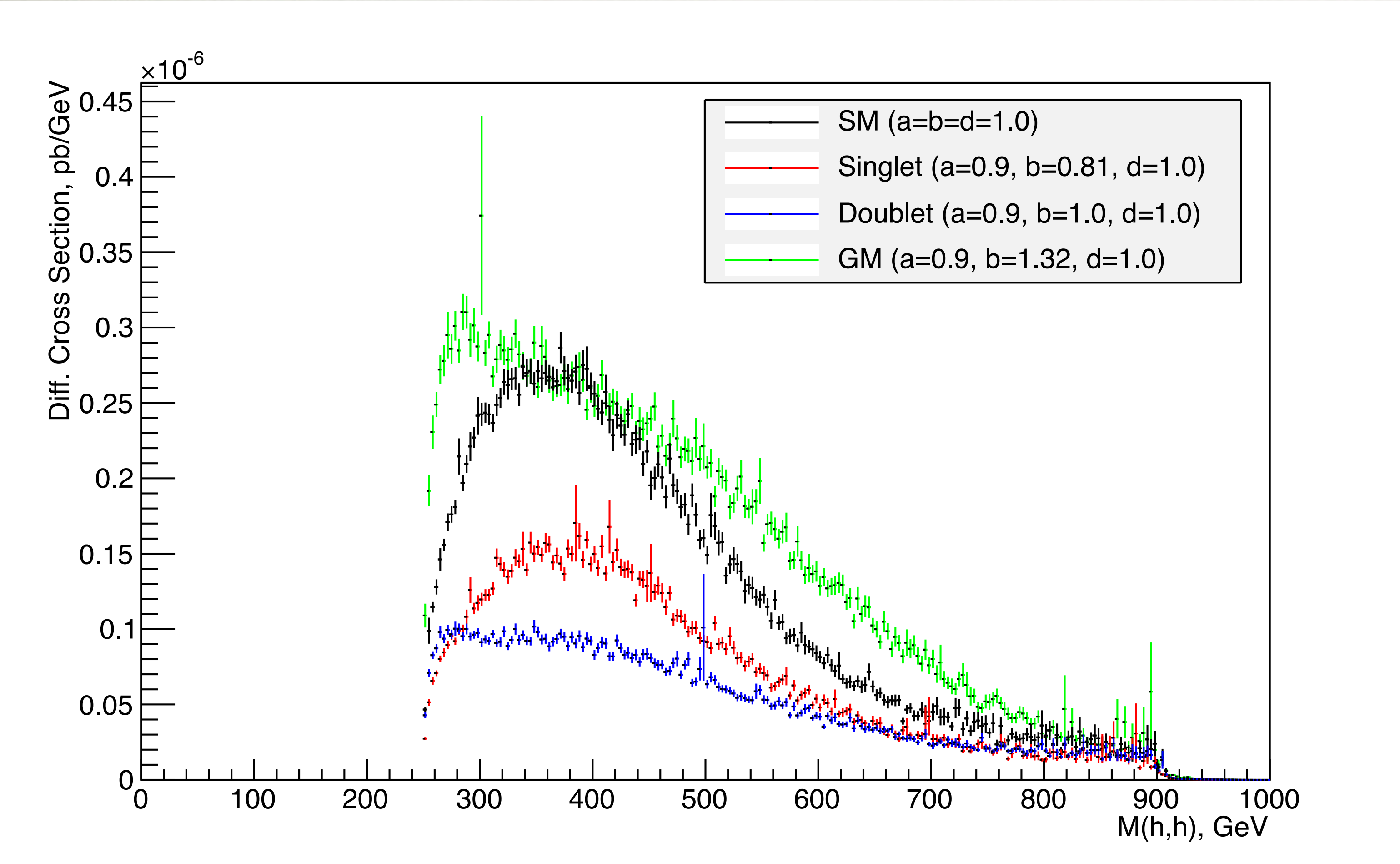


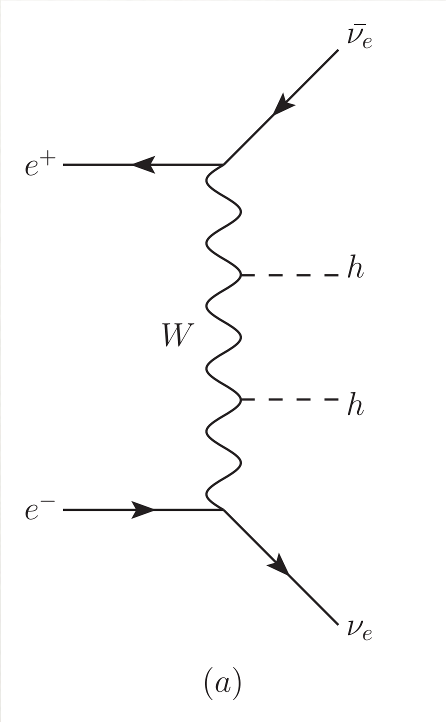


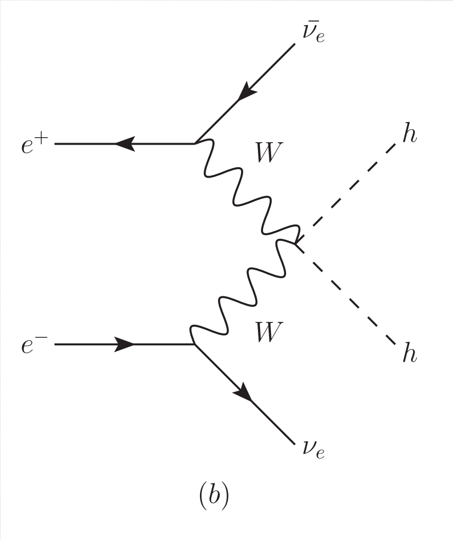


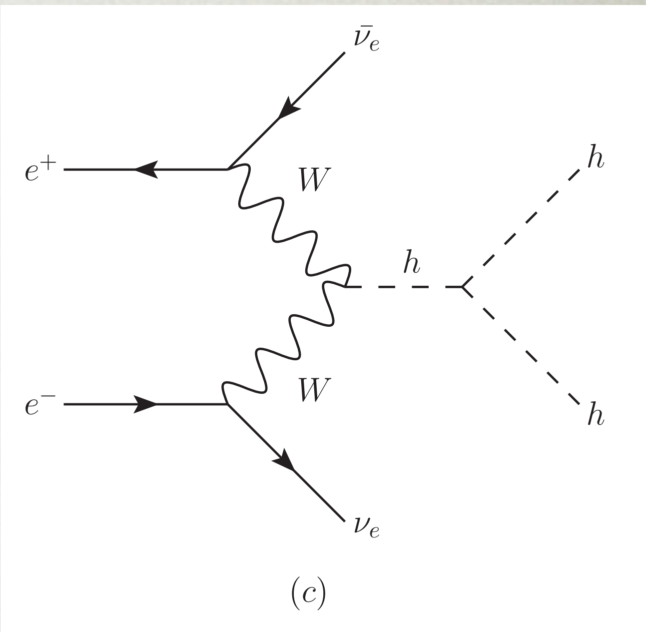
- Benchmark Models differ just in value of *b*
- Dig. (b) and (c) interfere constructively leading to enhancement at lower M_{hh} for larger *b* values (GM or Doublet model vs Singlet)

WBF at 1 TeV





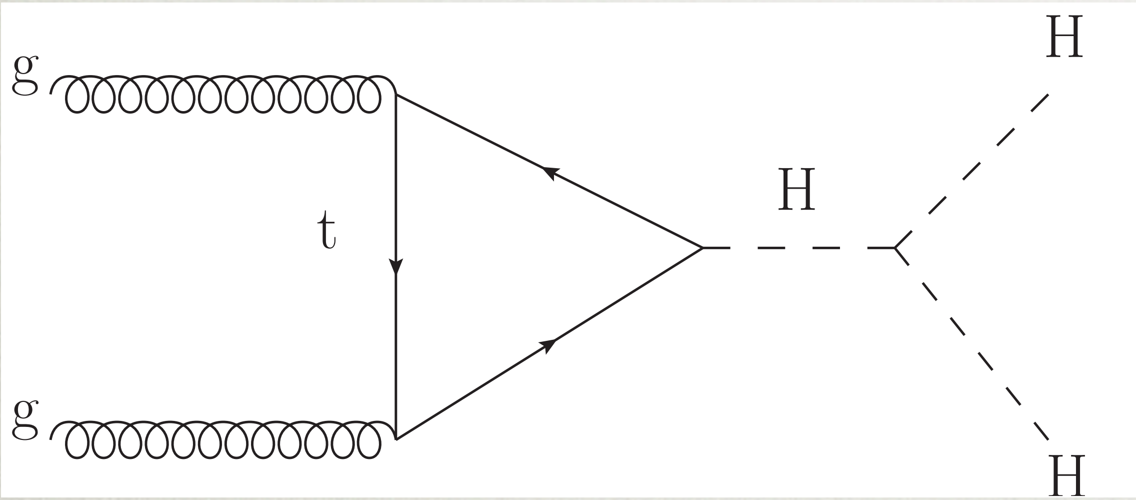


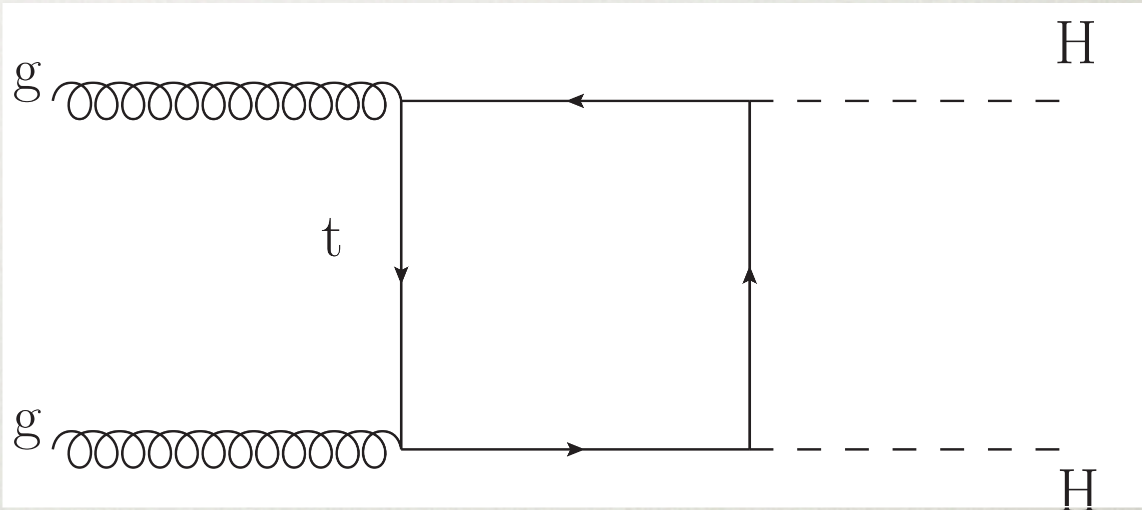


- Dig. (a) and (b) interfere destructively
- Leads to a flatter spectrum at intermediate M_{hh} for large b values (Doublet vs Singlet model)

Constraints on *d* from LHC

- Accessed via di-Higgs production through gluon fusion
- Depends on top Yukawa coupling as well as new particles in the gluon-fusion loop





figs. from F. Goertz, A. Papaefstathiou, L.L.Yang, J. Zurita [arXiv : 1301.3492]

Constraints on *d* from LHC

- di-Higgs production at LHC is not very sensitive to *b* (the hhVV coupling modification)
- *d* can be constrained to be +ve at 96% CL using 600 inv. fb at 14 TeV LHC

F. Goertz et al. [arXiv : 1301.3492]

- With 3000 inv. fb the 1 sigma uncertainty is reduced to +30% and -20%
- The study assumed $c = 1$ and no new particles in the loop
- A joint analysis from LHC and ILC data can thus be used to constrain *b*, *d* and new colored particles or higher-dimensional operators

Caveats/Viability of Benchmark Models & Methodology

Double Higgs production at ILC

- The approach we used to calculate di-Higgs rates doesn't account for contribution from t- and u-channel exchange of SU(2) triplet states in the doublet and GM model
- Doesn't include H -> h h where H is the heavier custodial singlet
- We assume these states are heavy enough to be kinematically forbidden at the 1 TeV ILC

$$M_{H^0} \gtrsim 910 \text{ GeV} \qquad e^+e^- \to Z(H \to hh)$$

$$M_{A^0} \gtrsim 875 \text{ GeV} \qquad e^+e^- \to h(A^0 \to Zh)$$

$$M_{H^\pm} \gtrsim 500 \text{ GeV} \qquad e^+e^- \to H^+H^-$$

For the Doublet case including these states increases the di-Higgs cross section by a few % for

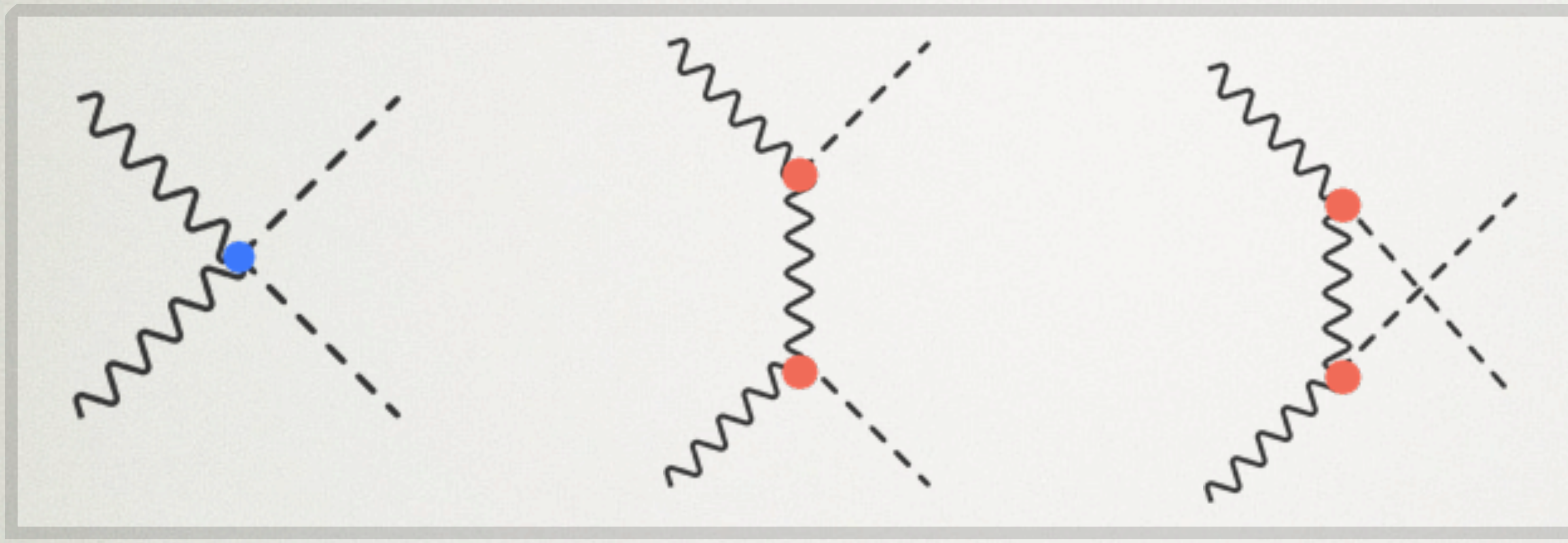
$$M_{H^+} = 660 \text{ GeV} \qquad M_{A^0} = 880 \text{ GeV}$$

Unitarity Constraints on Heavy States

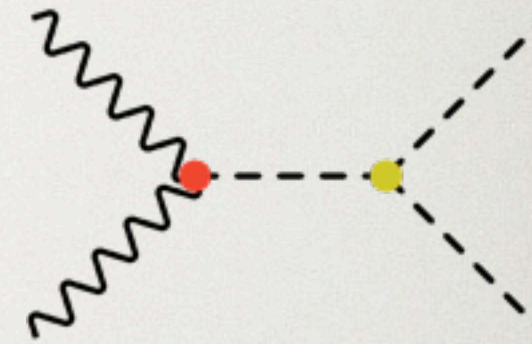
- We cannot assume the heavier states to be arbitrarily heavy
- This is because in the presence of Higgs coupling deviations we need contributions from NP to ensure perturbative unitarity
- $V_L V_L \to hh$
- $V_L V_L \to V_L V_L$
- We calculate these amplitudes at tree level and impose the following condition on the zeroth partial wave amplitude

$$|\mathrm{Re}(a_0)| \leq \frac{1}{2}$$

$V_L V_L \to hh$



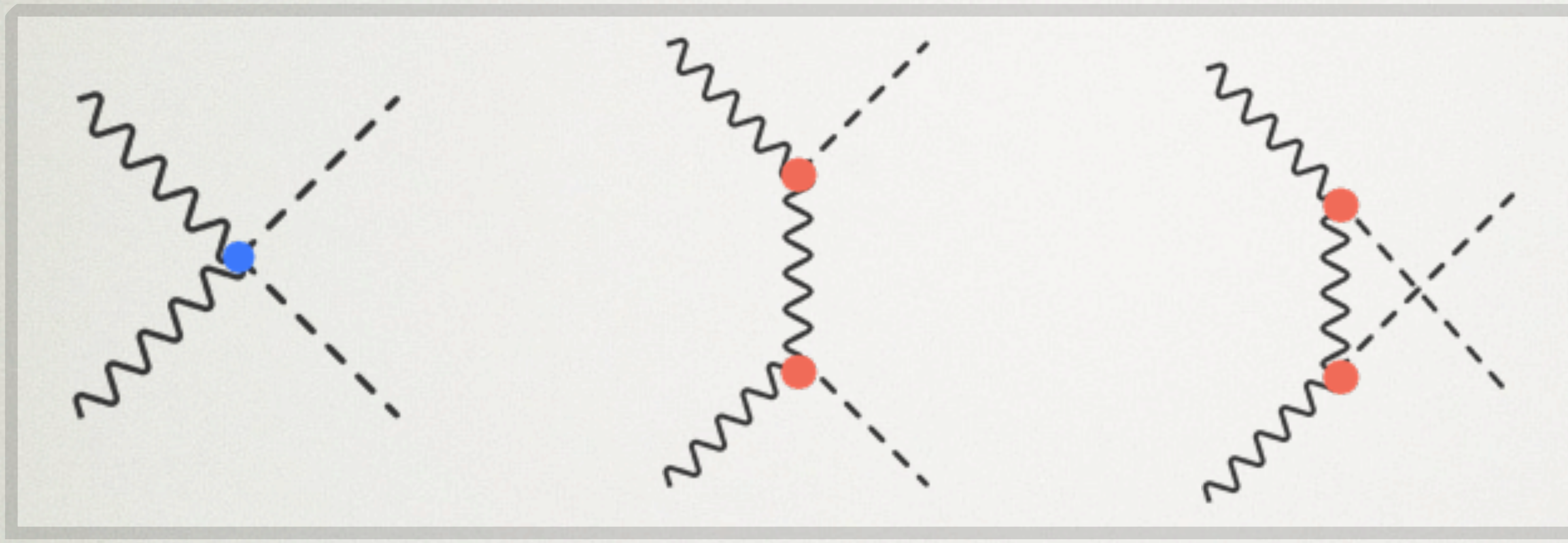
$\mathcal{O}\left(\frac{E^2}{v^2}\right)$



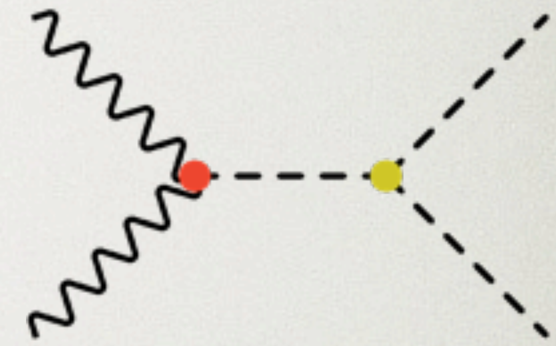
$\mathcal{O}\left(E^0\right)$

- Clearly the t- and u-channel exchange is required to restore unitarity when $b - a^2 \neq 0$

$V_L V_L \to hh$



$\mathcal{O}\left(\frac{E^2}{v^2}\right)$



$\mathcal{O}\left(E^0\right)$

Including SU(2) triplet state $(H^{\pm}, A^0)$ contributions we get

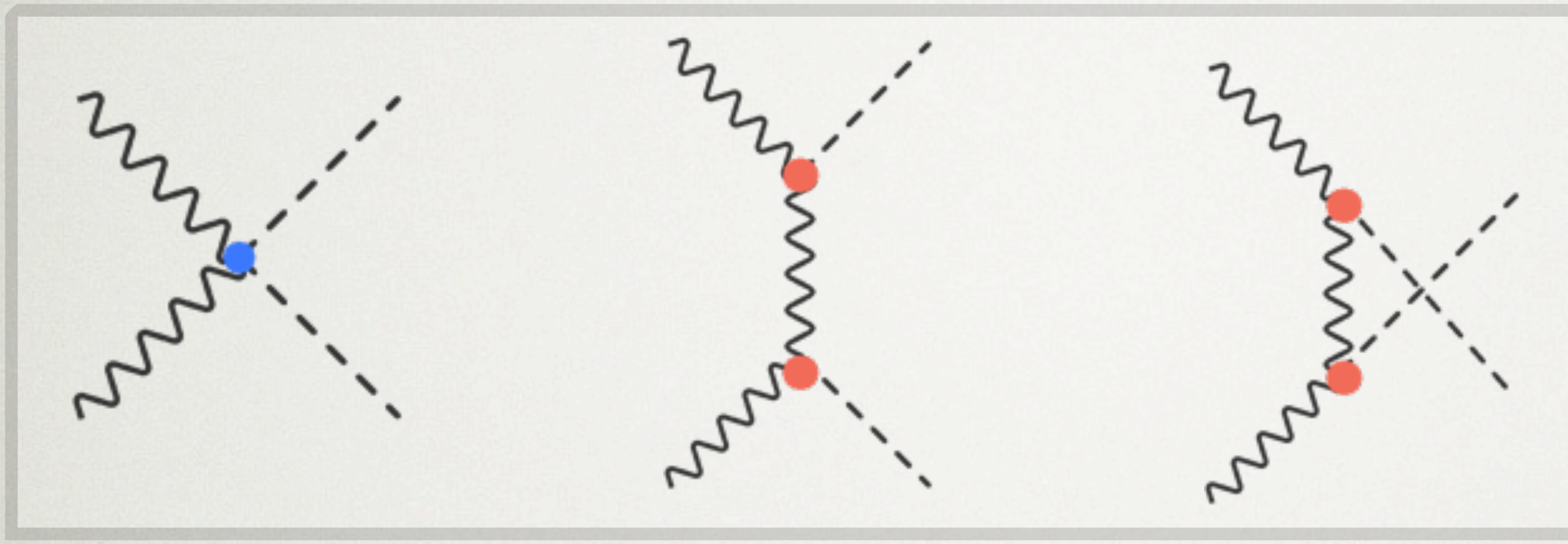
$$\text{Doublet}: \quad m^2_{H^{\pm},A^0} \lesssim \frac{8\pi v^2}{\sqrt{3}(1-a^2)} \simeq (2150\ \text{GeV})^2$$

a=0.9

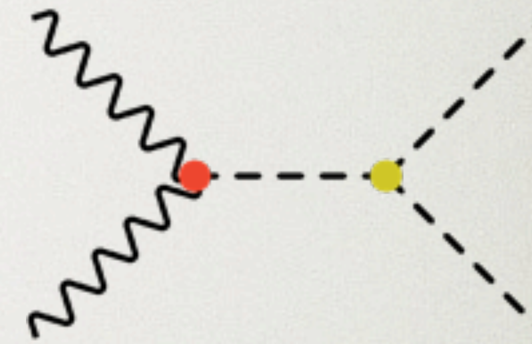
$$\text{GM}: \quad m^2_{H_3^{\pm},A_3^0} \lesssim \frac{3\pi v^2}{\sqrt{3}(1-a^2)} \simeq (1320\ \text{GeV})^2$$

Assuming $4\pi v^2_{\text{SM}} \gg m_h^2, m_W^2$ and triplet masses are degenerate

$V_L V_L \to hh$



$\mathcal{O}\left(\frac{E^2}{v^2}\right)$



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$$\text{GM}: \; m^2_{H_3^{\pm},A_3^0} \lesssim \frac{3\pi v^2}{\sqrt{3}(1-a^2)} \simeq (1320 \text{ GeV})^2$$

a=0.9

Coefficients are different because the triplet states in these models have different EW quantum numbers

$V_L V_L \to V_L V_L$

$\mathcal{O}\left(\frac{E^4}{v^4}\right)$ $\mathcal{O}\left(\frac{E^4}{v^4}\right)$ $\mathcal{O}\left(\frac{E^4}{v^4}\right)$ $\mathcal{O}\left(\frac{E^2}{v^2}\right)$ $\mathcal{O}\left(\frac{E^2}{v^2}\right)$

$\mathcal{O}\left(\frac{E^2}{v^2}\right)$

Coupled channel analysis yields

$$m_{H^0}^2 \lesssim \frac{16\pi v_{\rm SM}^2}{5(1-a^2)} \simeq (1790 \text{ GeV})^2 \longleftarrow \boxed{\text{a=0.9}}$$

- Thus perturbative unitarity constraints do not prevent us from assuming $H^0 \to hh$ contributions are beyond the kinematic reach of the 1 TeV ILC

$V_L V_L \to V_L V_L$

$\mathcal{O}\left(\frac{E^4}{v^4}\right)$ $\mathcal{O}\left(\frac{E^4}{v^4}\right)$ $\mathcal{O}\left(\frac{E^4}{v^4}\right)$ $\mathcal{O}\left(\frac{E^2}{v^2}\right)$ $\mathcal{O}\left(\frac{E^2}{v^2}\right)$

$\mathcal{O}\left(\frac{E^2}{v^2}\right)$

$$m_{H^0}^2 \lesssim \frac{16\pi v_{\mathrm{SM}}^2}{5(1-a^2)} \simeq (1790 \text{ GeV})^2 \longleftarrow \boxed{a=0.9}$$

- We neglect exchange of custodial 5-plet as its contribution is small for small v_χ

$a \neq c$

- Benchmark Models assumed Higgs mixing with a scalar that doesn't participate in EWSB or break custodial SU(2)
- There are well motivated models for which these assumptions do not hold
- If new scalar participates in EWSB then $a \neq c$
- This can be determined from the high precision measurements of single Higgs couplings at the ILC

$a \neq c$

- The extraction of *b* and *d* is not affected
- What changes is the interpretation of *a* and *c*
- *a* and *c* can be used to extract the mixing angle and scalar vevs for an assumption of EW quantum numbers
- this leads to a prediction of *b* for the chosen model

$$a_V = \cos\theta \sin\beta - \sqrt{b_V^\chi} \sin\theta \cos\beta.$$

$$c = \frac{\cos\theta}{v_\phi / v_{\rm SM}} = \frac{\cos\theta}{\sin\beta}.$$

$$b_V = \cos^2\theta + b_V^\chi \sin^2\theta.$$

SM + Doublet

$$a_V = \cos\theta \sin\beta - \sqrt{b_V^{\chi}} \sin\theta \cos\beta, \qquad c = \frac{\cos\theta}{\sin\beta} \qquad \sin\beta = v_1/v_{\mathrm{SM}}$$

$$\tan\beta' = \frac{v_2}{v_1} \qquad \cos\theta = \sin(\beta' - \alpha)$$

Softly broken Z_2

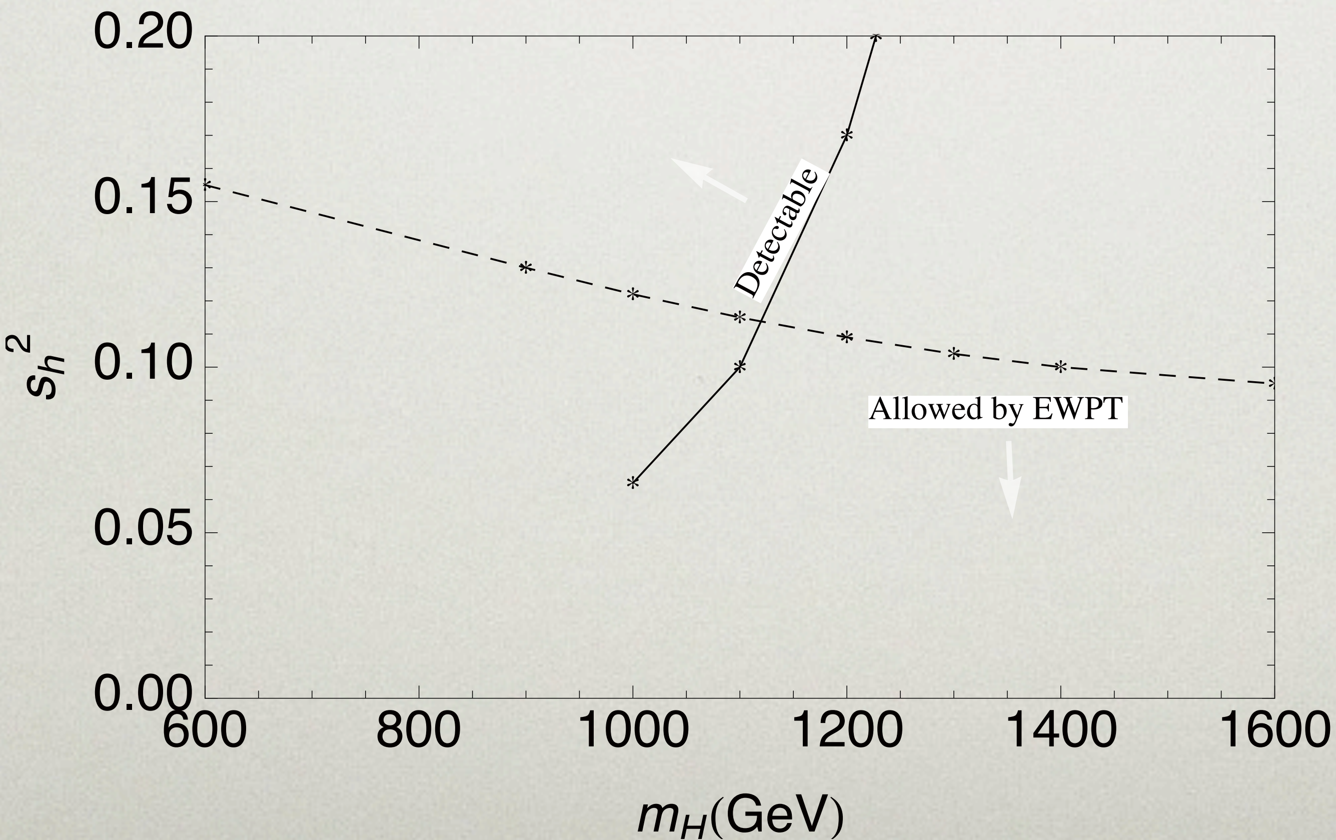
- enforces fermion coupling structure
- additional doublet zero vev $\quad a = c = 1$
- additional doublet non zero vev $\quad a \neq c$

No Z_2

- additional doublet zero vev $\quad a = c \neq 1$
- Requires a theory of flavor to explain absence of FC neutral Higgs couplings

Sufficient Mixing & EW Precision - BM I

- Can we get the mixing we require for each of our benchmark models?
- For a mixed-in scalar singlet dimensionful coupling allows for enough mixing without needing large quartic scalar couplings

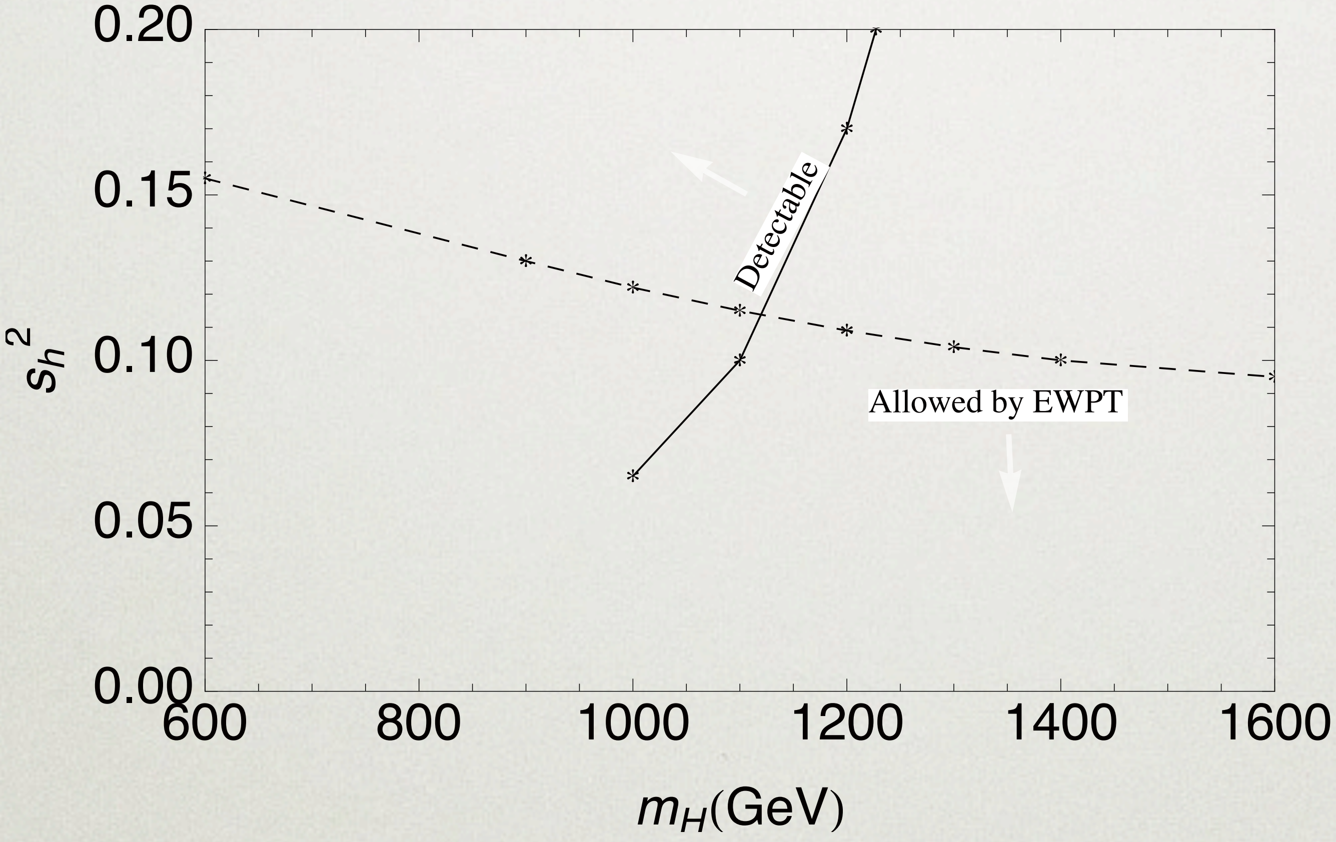


R. Gupta, J.Wells, H. Rhezhak, arXiv : 1206.3560

Sufficient Mixing & EW Precision - BM I

- Main constraint from EW precision observables

$$S = \cos^2\theta\, S_{\rm SM}(m_h) + \sin^2\theta\, S_{\rm SM}(m_{H^0})$$

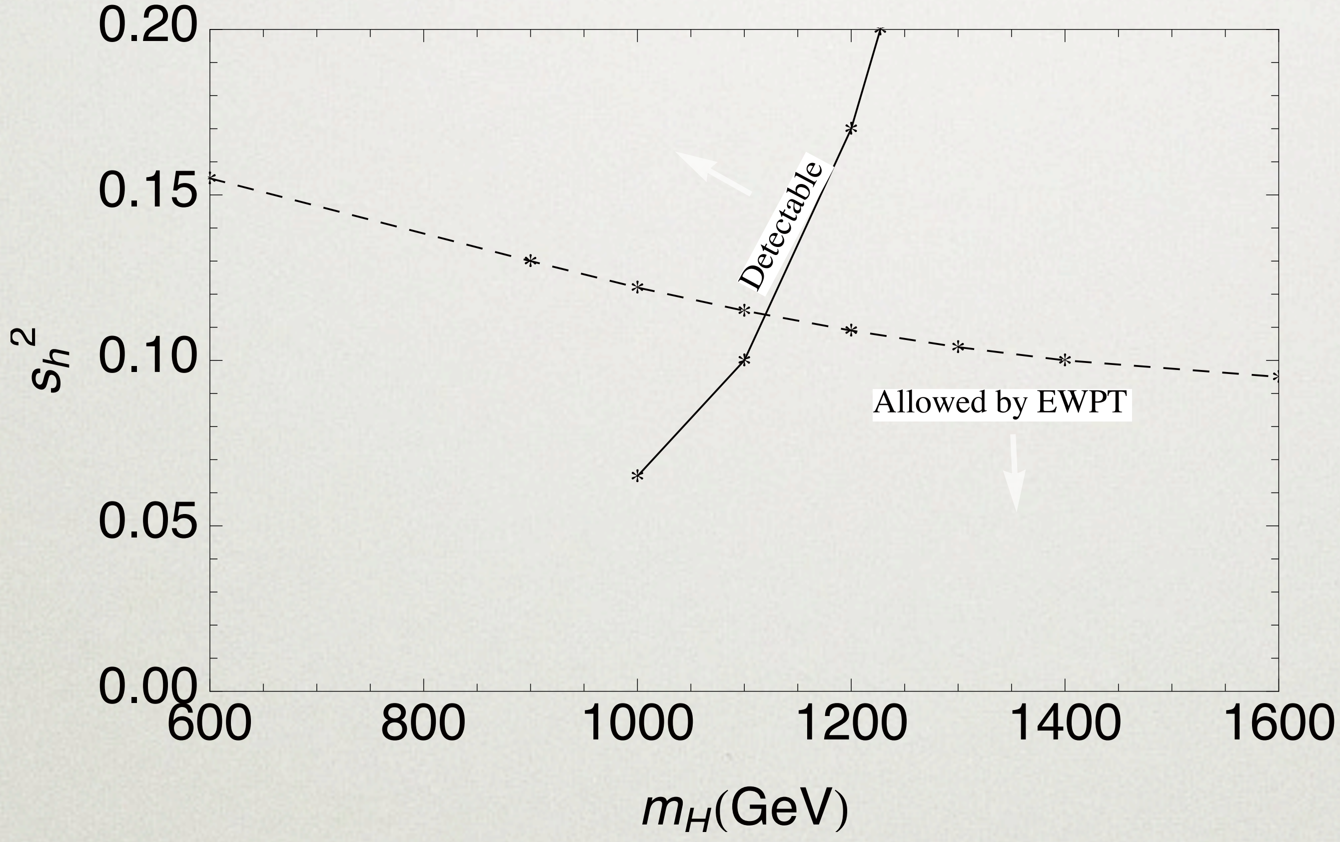


R. Gupta, J.Wells, H. Rhezhak, arXiv : 1206.3560

- For most of the heavy scalar mass range (910 - 1790 GeV) $\sin^2\theta \lesssim 0.1$

Sufficient Mixing & EW Precision - BM I

$\sin^2\theta = 0.19$ represents a mild violation which can be compensated by additional new physics that adjusts the S and T parameters



R. Gupta, J.Wells, H. Rhezhak, arXiv : 1206.3560

Note that a smaller mixing corresponding to a = 0.95 would be allowed by EWPT

Sufficient Mixing & EW Precision - BM II

Harder to obtain mixing while keeping the additional states beyond the kinematic reach of the 1 TeV ILC

We use 2HDMC to scan the general CP-conserving potential

$$\begin{aligned}\mathcal{V}_{\rm gen} =& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \\ &+ \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ &+ \left\{\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\}.\end{aligned}$$

$\lambda_6 = \lambda_7 = 0$

Obtaining sufficient mixing $\cos\theta \equiv \sin(\beta - \alpha) = 0.9$ requires large quartics λ_3 and λ_4 of order 10

These quartics lead to a mass splitting between charged scalars and the pseudoscalar and therefore to the *T* parameter

Could be compensated for by isospin-violating new physics

Sufficient Mixing & EW Precision - BM III

Most general custodial SU(2) preserving potential

$$
\begin{aligned}
V = &\frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 \\
&+\lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) + \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) \\
&+\lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\
&+M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(X)_{ab} \\
&+M_2\mathrm{Tr}(X^\dagger t^a X t^b)(X)_{ab},
\end{aligned}
$$

The two dimensionful parameters M1 and M2 allow for large masses and sufficient mixing without quartics becoming large

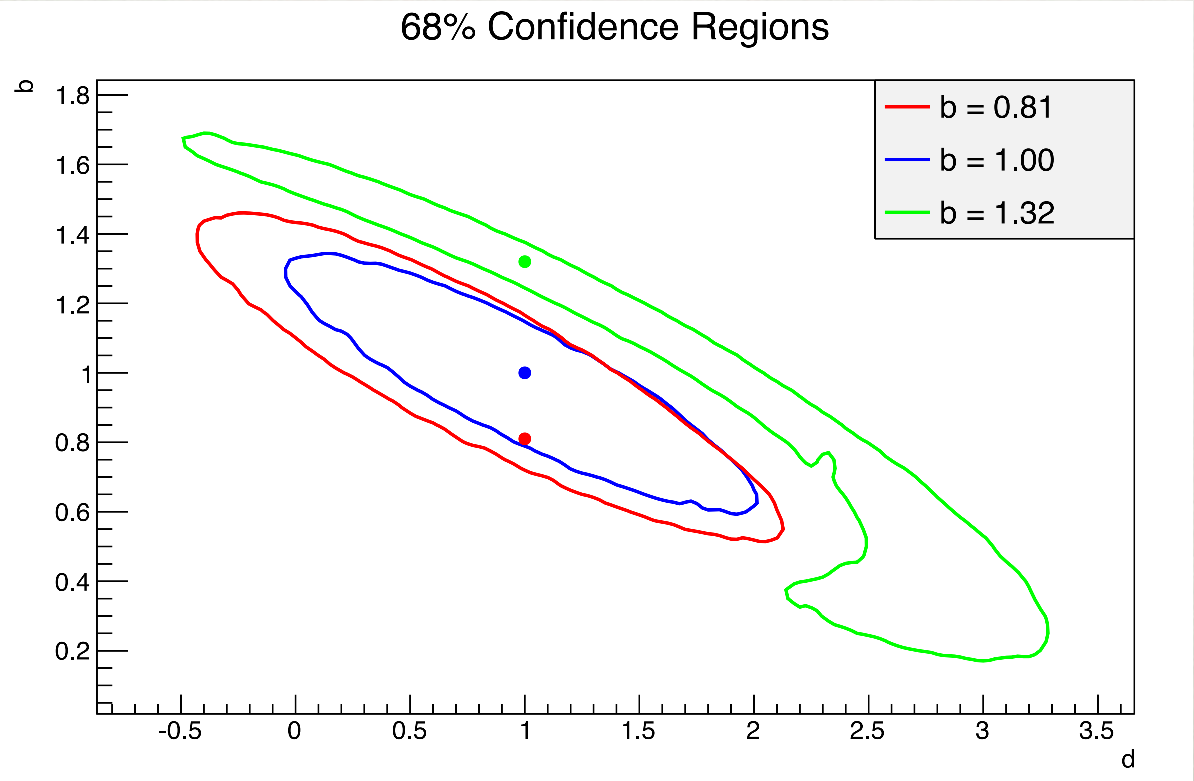
Traditionally these terms are omitted by imposing the discrete symmetry $X \to -X$

We find obtain $\cos\theta = 0.9$ with M1 ~ -2400 GeV and $v_\chi \sim 30$ GeV

SM + Septet (T=3,Y=4)

$$b_W^\chi = b_Z^\chi = 16$$

- For a = 0.9 and small septet vev we get b = 3.85
- This will be well separated from the 3 BM models
- Even for a = 0.99, the septet yields a sizable b = 1.3



Broken Custodial SU(2)

- If custodial SU(2) is broken then $b_W \neq b_Z$
- Two measurements will not be sufficient to measure b_W, b_Z and d
- Additional information from kinematic discriminants and/or the LHC will be needed

Conclusions

- If Higgs couplings show deviations from the SM expectation a direct measurement of the $hhVV$ would be important to determine EW quantum numbers of the other scalar
- This measurement is extremely difficult at the LHC
- At the ILC it is accessible via di-Higgs production
- di-Higgs production has mainly been studied as a handle on the triple-Higgs coupling

Conclusions

- The *hhVV* can be separated from the *hhh* coupling with rate measurements at two different centre of mass energies
- In addition LHC measurements can constrain the *hhh* independently

BACKUP SLIDES

Unitarity limits

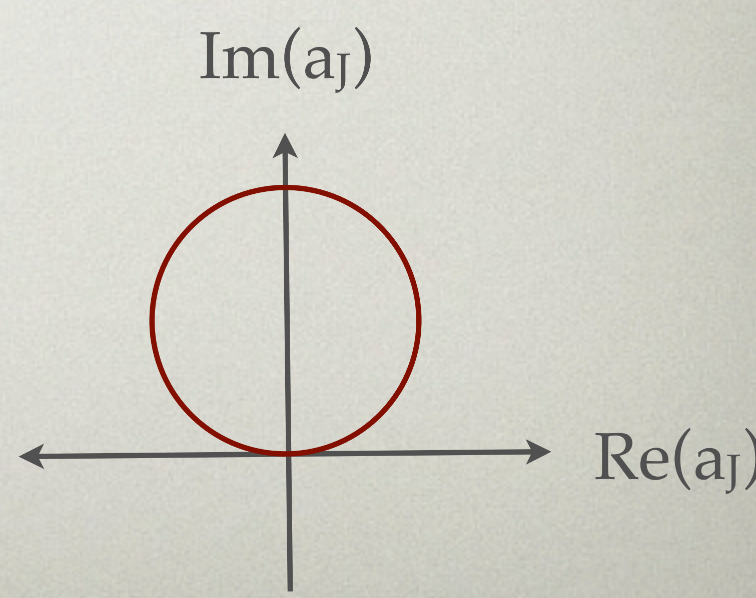
The amplitude for a scattering process can be written in terms of partial wave amplitudes of definite angular momentum

$$\mathcal{M} = 16\pi \sum_J (2J+1) a_J P_J(\cos\theta)$$

The cross section in each partial wave is limited

Since this is a result of the unitarity of the S-matrix the bounds on the partial wave amplitudes are called unitarity limits

$$|\mathrm{Re}(a_J)| \leq \frac{1}{2}$$



di-Higgs from VBF

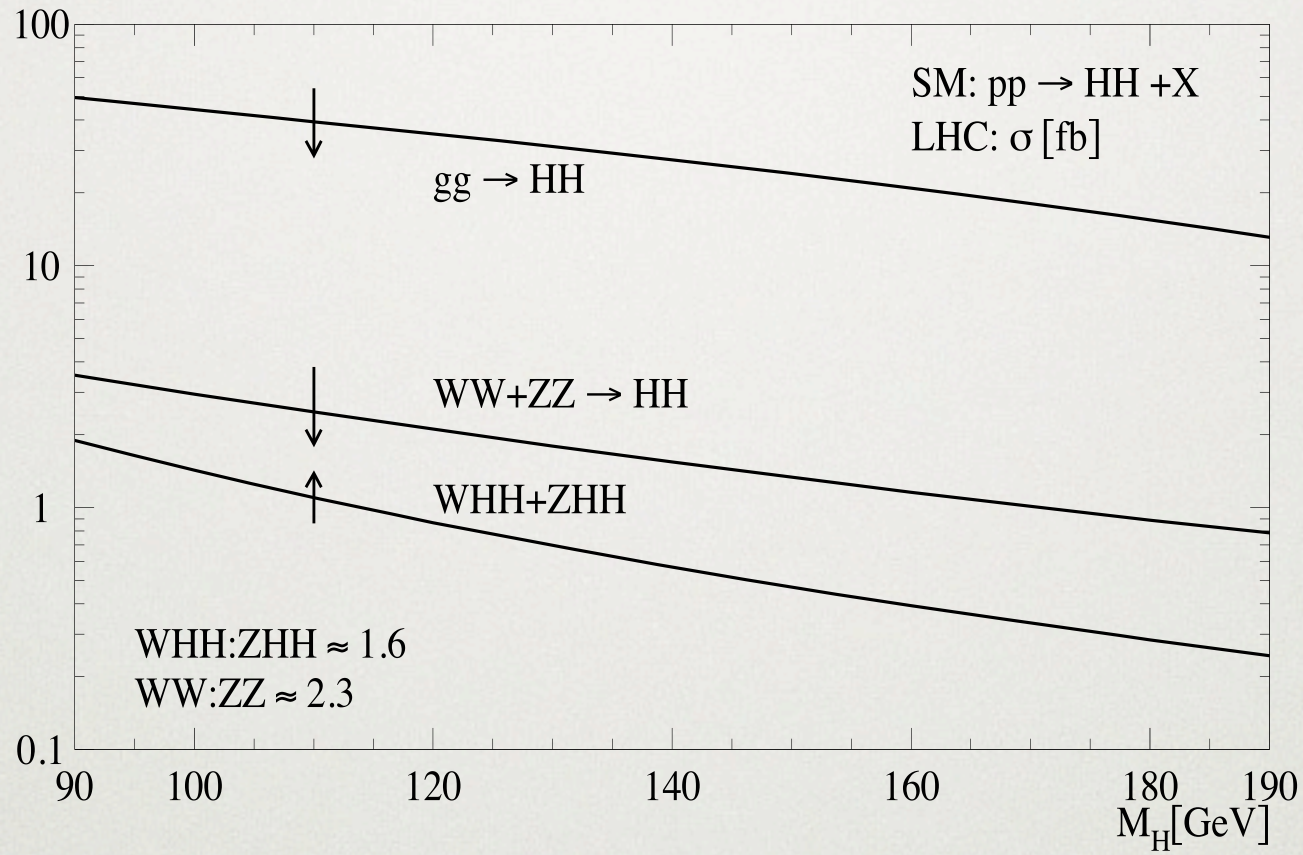


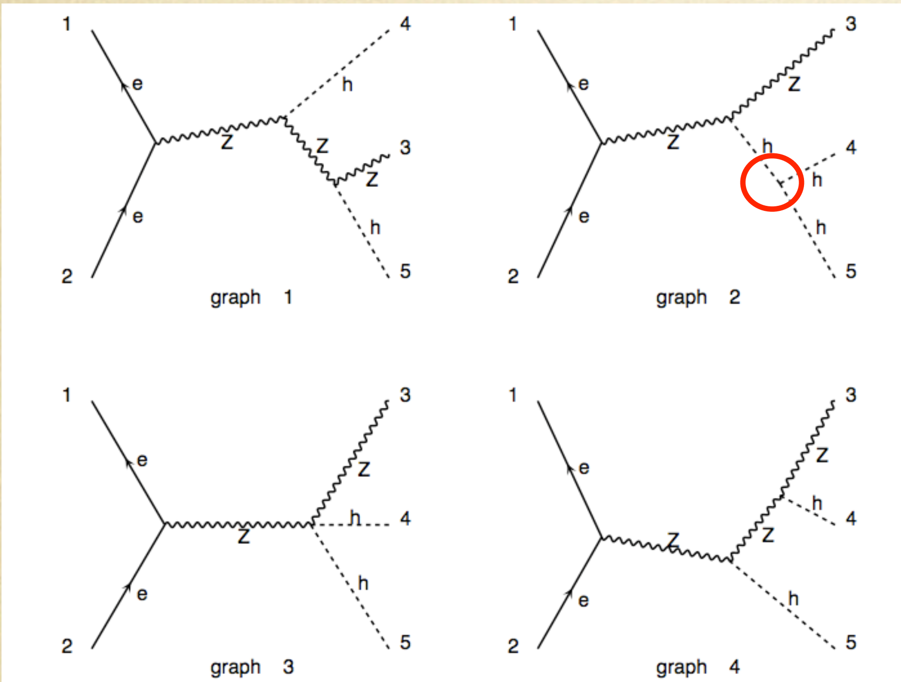
Figure 3.40: The cross sections for gluon fusion, $gg \to HH$*, the* WW/ZZ *fusion* $qq \to qqWW/ZZ \to HH$ *and the double Higgs–strahlung* $q\bar{q} \to WHH + ZHH$ *in the SM as a function of* M_H*. The vertical arrows correspond to a variation of the trilinear Higgs coupling from* $\frac{1}{2}$ *to* $\frac{3}{2}$ *of the SM value,* $\lambda'_{HHH} = 3M_H^2/M_Z^2$*; from Ref. [254].*

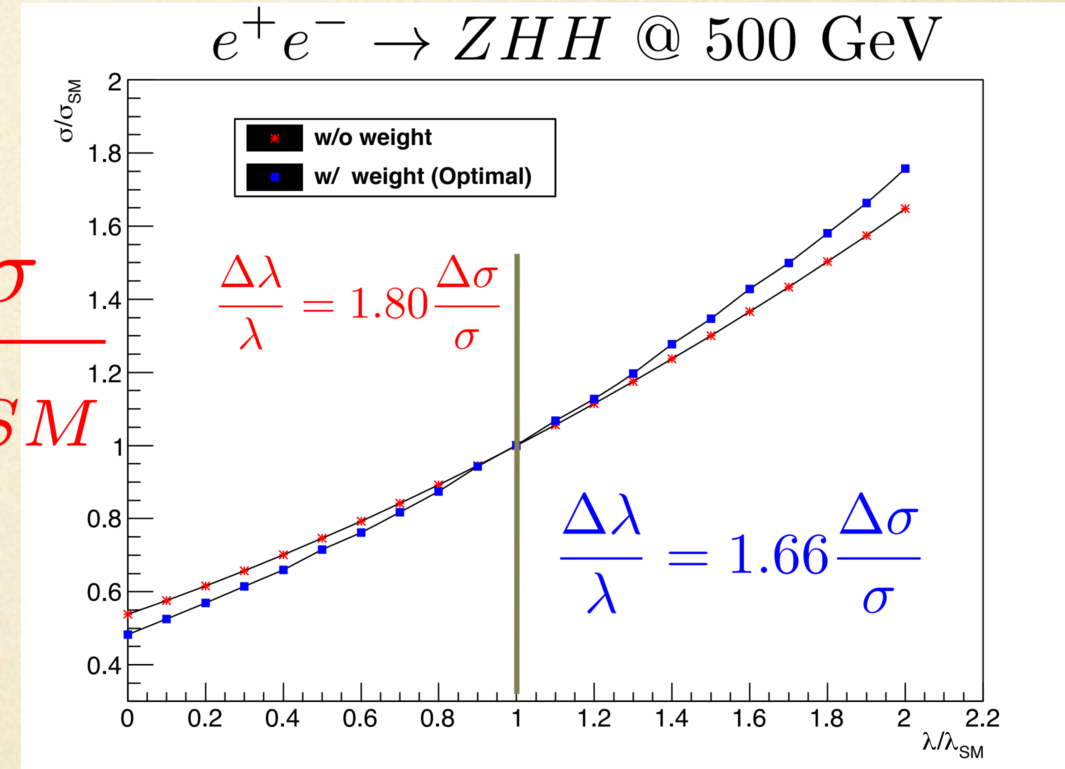
Major Processes

Energy	Reaction	Physics Goal	Polarization
91 GeV	$e^+e^- \to Z$	ultra-precision electroweak	**A**
160 GeV	$e^+e^- \to WW$	ultra-precision W mass	**H**
250 GeV	$e^+e^- \to Zh$	precision Higgs couplings	**H**
350–400 GeV	$e^+e^- \to t\bar{t}$	top quark mass and couplings	**A**
	$e^+e^- \to WW$	precision W couplings	**H**
	$e^+e^- \to \nu\bar{\nu}h$	precision Higgs couplings	**L**
500 GeV	$e^+e^- \to f\bar{f}$	precision search for Z'	**A**
	$e^+e^- \to t\bar{t}h$	Higgs coupling to top	**H**
	$e^+e^- \to Zhh$	Higgs self-coupling	**H**
	$e^+e^- \to \tilde{\chi}\tilde{\chi}$	search for supersymmetry	**B**
	$e^+e^- \to AH, H^+H^-$	search for extended Higgs states	**B**
700–1000 GeV	$e^+e^- \to \nu\bar{\nu}hh$	Higgs self-coupling	**L**
	$e^+e^- \to \nu\bar{\nu}VV$	composite Higgs sector	**L**
	$e^+e^- \to \nu\bar{\nu}t\bar{t}$	composite Higgs and top	**L**
	$e^+e^- \to \tilde{t}\tilde{t}^*$	search for supersymmetry	**B**

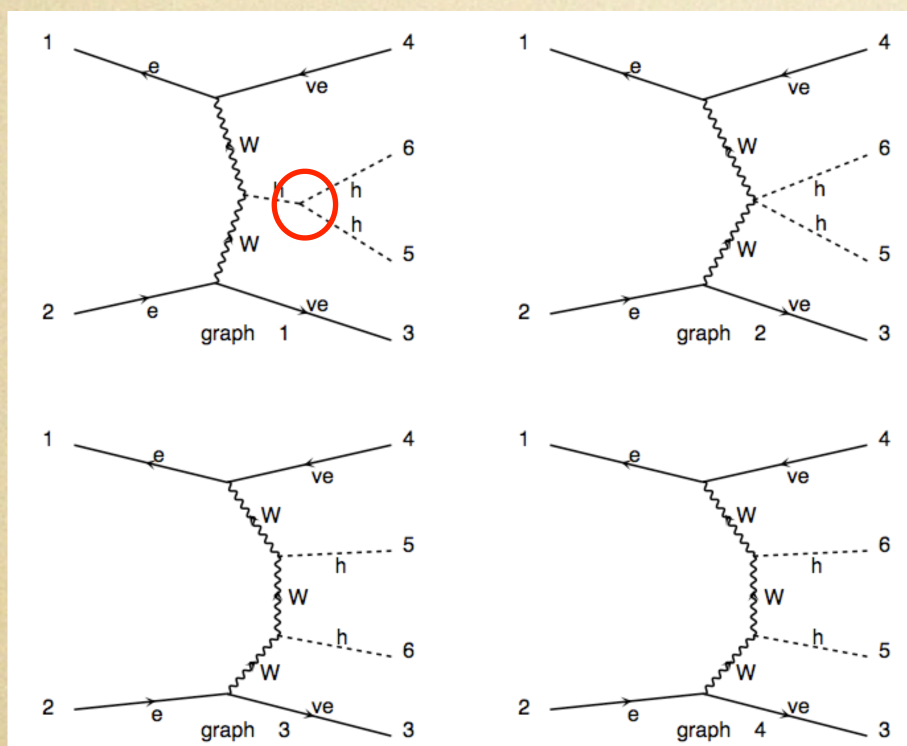
General issue: sensitivity to the cross section

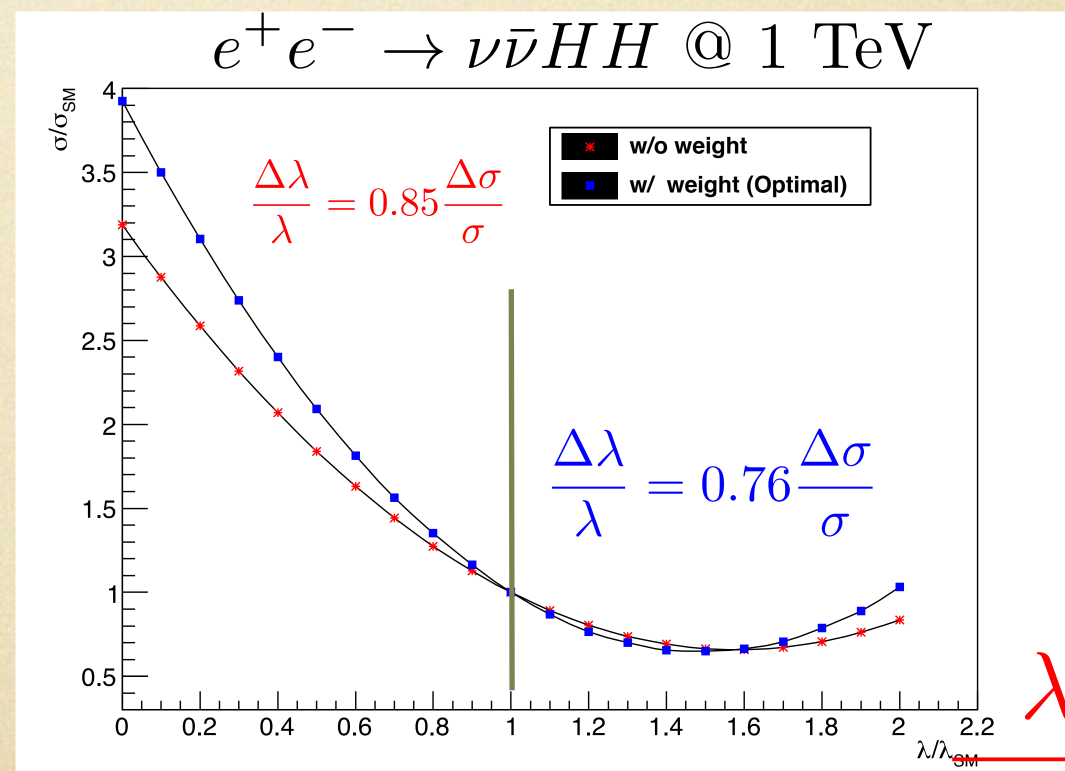
effect of irreducible diagrams





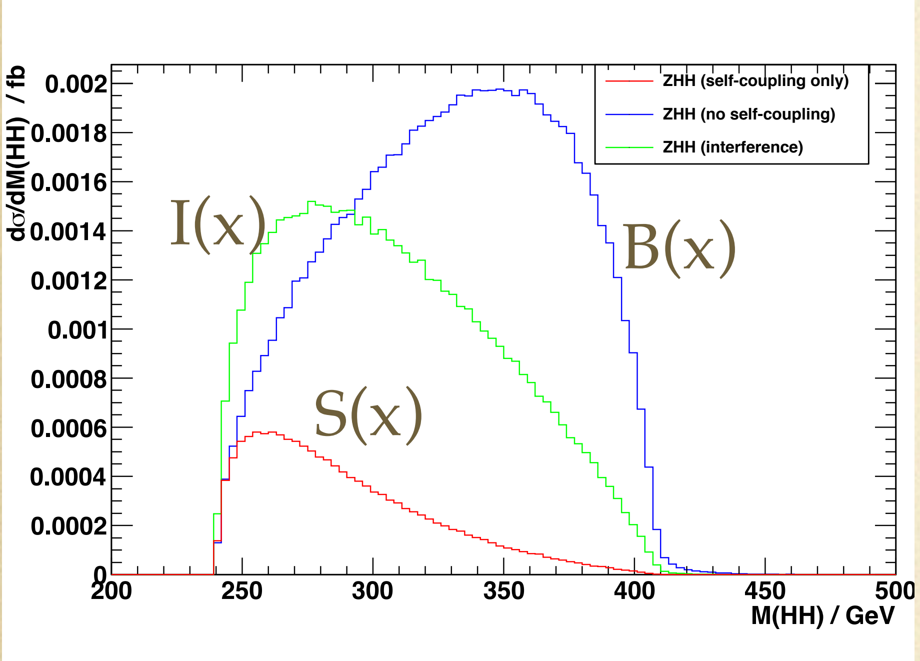
$$\frac{\sigma}{\sigma_{SM}}$$





$$\frac{\lambda}{\lambda_{SM}}$$

weighting method to enhance the coupling sensitivity

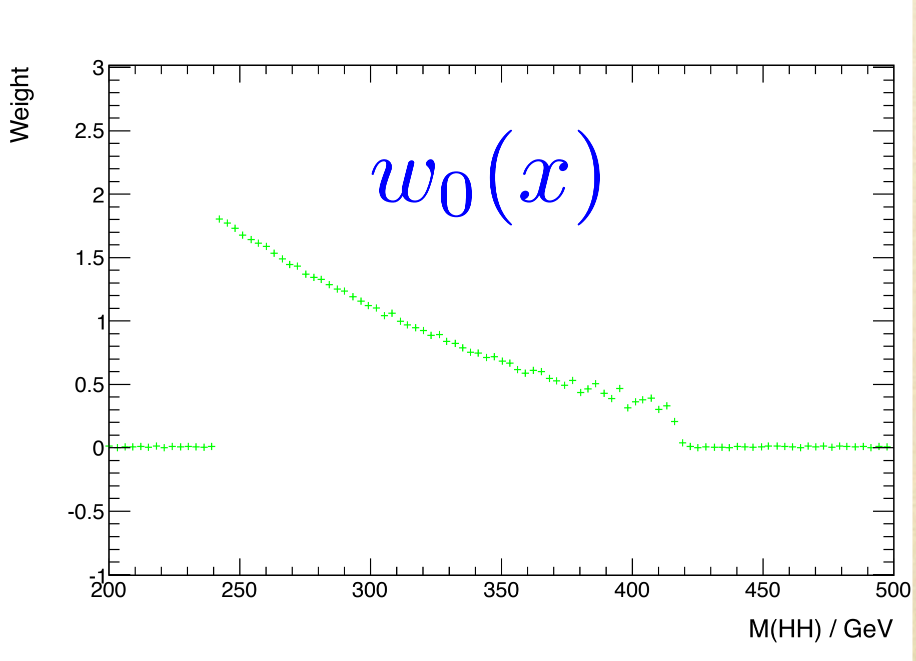


$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = B(x) + \lambda I(x) + \lambda^2 S(x)$$

irreducible interference self-coupling

observable: weighted cross-section

$$\sigma_w = \int \frac{\mathrm{d}\sigma}{\mathrm{d}x} w(x)\mathrm{d}x$$



equation of the optimal w(x):

$$\sigma(x)w_0(x)\int (I(x)+2S(x))w_0(x)\mathrm{d}x = (I(x)+2S(x))\int \sigma(x)w_0^2(x)\mathrm{d}x$$

general solution:

$$w_0(x) = c \cdot \frac{I(x)+2S(x)}{\sigma(x)}$$

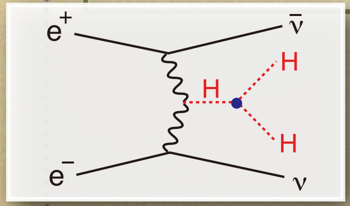
c: arbitrary normalization factor

Higgs Self-coupling: ILC 1TeV: Full Sim.

signal and backgrounds (reduction table)

Polarization: (e-,e+)=(-0.8,+0.2) $E_{cm} = 1$ TeV, $M_H = 120$ GeV $\int L = 2\ ab^{-1}$

	Expected	Generated	pre-selction	cut1	cut2	cut3	cut4
ννhh (WW F)	272	1.05×10⁵	127	107	77.2	47.6	35.7
ννhh (ZHH)	74.0	2.85×10⁵	32.7	19.7	6.68	4.88	3.88
ννbbbb	650	2.87×10⁵	553	505	146	6.21	4.62
ννccbb	1070	1.76×10⁵	269	242	63.3	2.69	0.19
yyxyyx	3.74×10⁵	1.64×10⁶	18951	4422	38.5	26.7	1.83
yyxyeν	1.50×10⁵	6.21×10⁵	812	424	44.4	11.0	0.73
yyxylν	2.57×10⁵	1.17×10⁶	13457	4975	202	84.5	4.86
ννZH	3125	7.56×10⁴	522	467	257	30.6	17.6
BG	7.86×10⁵		34597	11054	758	167	33.7
significance	0.30		0.68	1.01	2.67	3.25	4.29



$$\frac{\Delta\sigma}{\sigma} \approx 23\% \qquad \frac{\Delta\lambda}{\lambda} \approx 20\% \to 18\%$$ with weighting

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ILD DBD Study (Junping Tian)

difficulties

fundamental:

- irreducible SM diagrams, significantly degrade the coupling sensitivity.
- very small cross section (σ_{ZHH}~0.22 fb with P_L) and we are only using ~40% of the signal (both H-->bb). large integrated luminosity needed. (high beam polarization helps a lot)
- huge SM background (tt/WWZ, ZZ/Zγ, ZZZ/ZZH), 3-4 orders higher.

technical:

- Higgs mass reconstruction: mis-clustering, missing neutrinos, wrong pairing.
- flavor tagging and isolated-lepton selection: need very high efficiency and purity.
- neural-net training: separate neural-nets, huge statistics needed.

developments since LoI time

- LCFIPlus: Vertexing before jet-clustering --> flavor tagging much improved
- Improved data selection (neural-net optimization)
- Event weighting to enhance the signal diagram

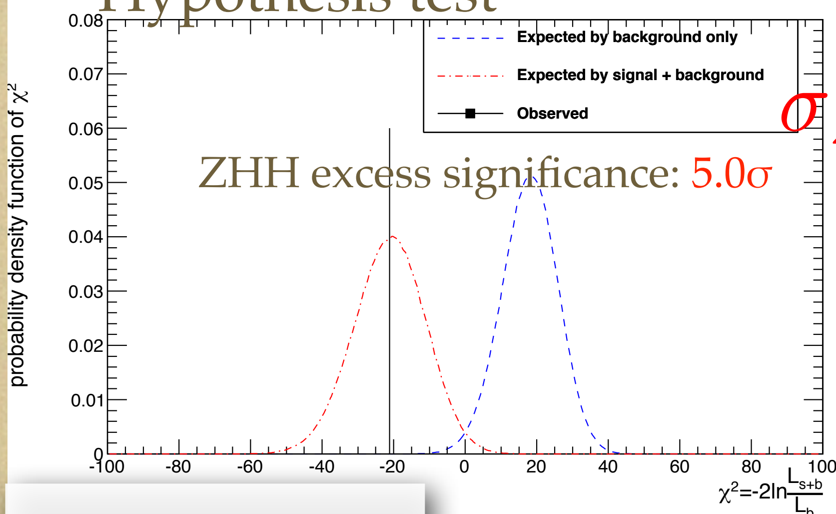
preliminary

DBD analysis at 500 GeV (combined)

P(e-,e+)=(-0.8,0.3) $e^+ + e^- \to ZHH$ $M(H) = 120\text{GeV}$ $\int Ldt = 2\text{ab}^{-1}$

Energy (GeV)	Modes	signal	background	significance	
				excess (I)	measurement (II)
500	$ZHH \to (l\bar{l})(b\bar{b})(b\bar{b})$	3.7	4.3	1.5σ	1.1σ
		4.5	6.0	1.5σ	1.2σ
500	$ZHH \to (\nu\bar{\nu})(b\bar{b})(b\bar{b})$	8.5	7.9	2.5σ	2.1σ
500	$ZHH \to (q\bar{q})(b\bar{b})(b\bar{b})$	13.6	30.7	2.2σ	2.0σ
		18.8	90.6	1.9σ	1.8σ

Hypothesis test

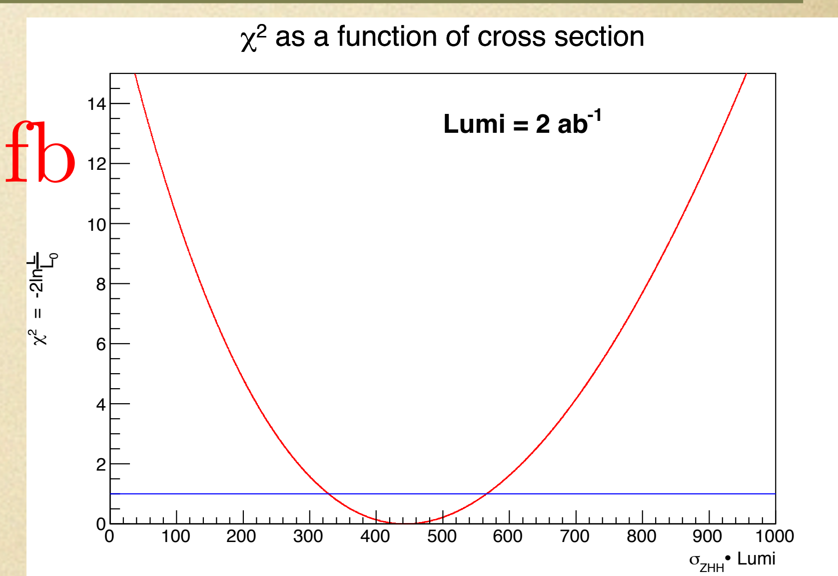


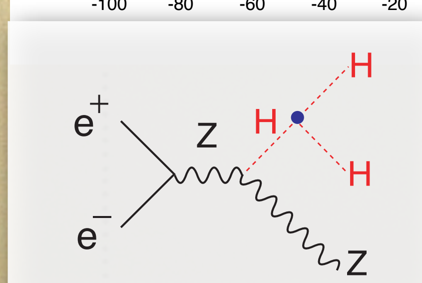
ZHH excess significance: 5.0σ

$$\sigma_{ZHH} = 0.22 \pm 0.06 \text{ fb}$$

$$\delta\sigma/\sigma = 27\%$$

$$\delta\lambda/\lambda = 48\%$$





(cf. 80% for qqbbbb at the LoI time)

with weighting, it would be: $\frac{\delta\lambda}{\lambda} = 44\%$

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